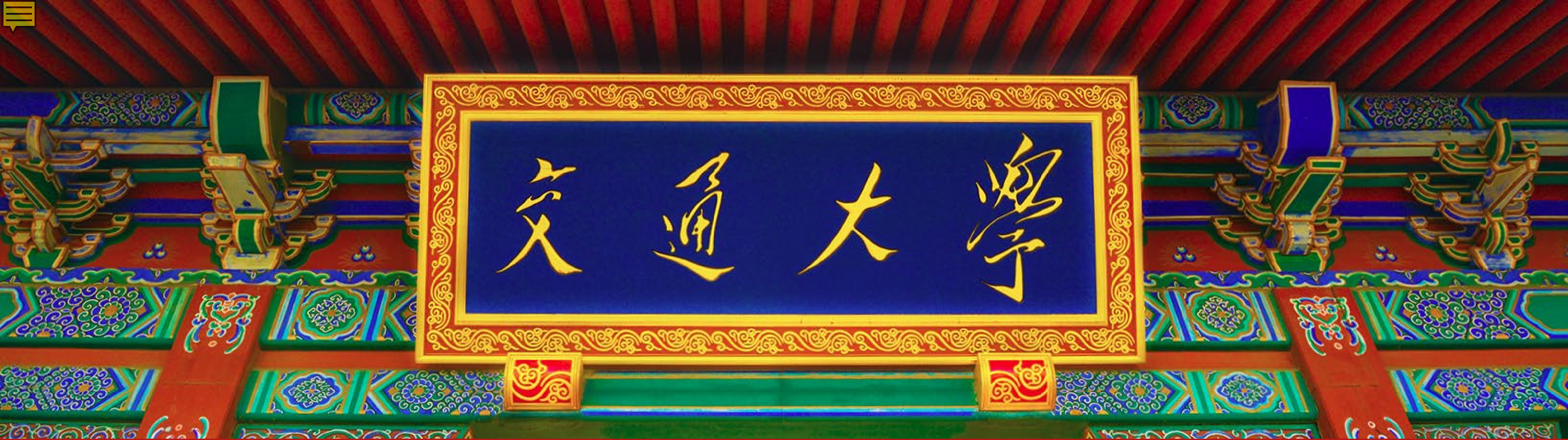

Explore fluid/wave patterns in ducts

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2020/11/23

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Outline

1 What is fluid/wave patterns? Why is it important?

2.1 Is all the emergence of a pattern predictable?

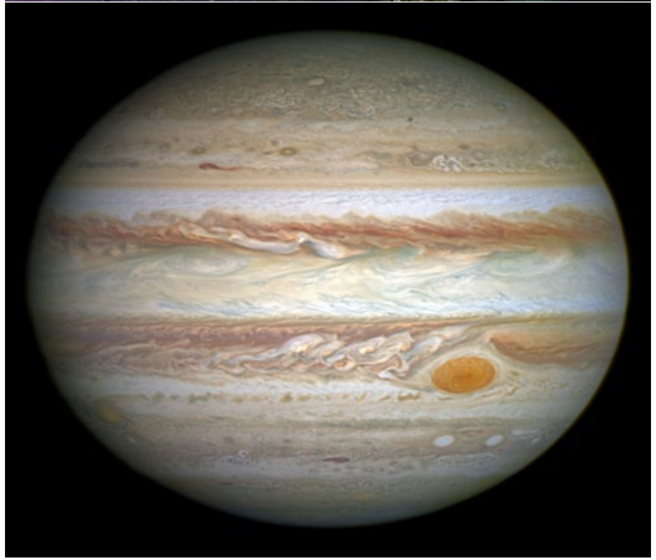
Rayleigh-Bénard instability , Taylor-Couette instability, Duct modes in swirl flow

2.2 How to use the patterns in real turbomachinery? Some Applications.

My experience of “patterns”

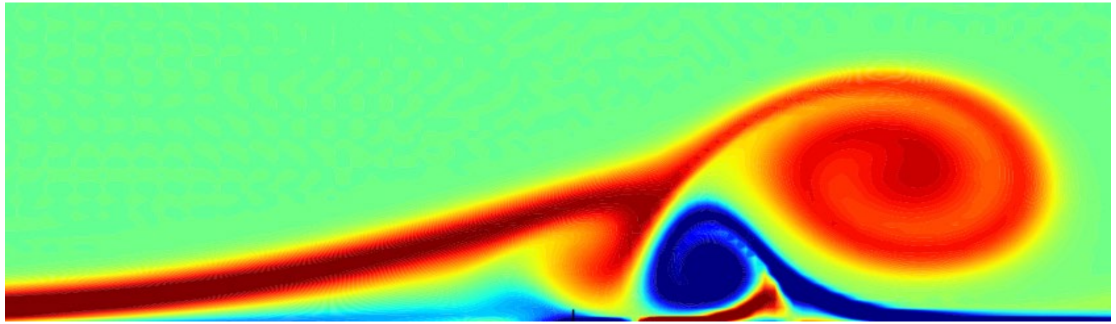
Matthew Colbrook Sheehan Olver
David Abrahams’s phd student
Anastasia Kisil & Matthew Priddin

Similarity of patterns

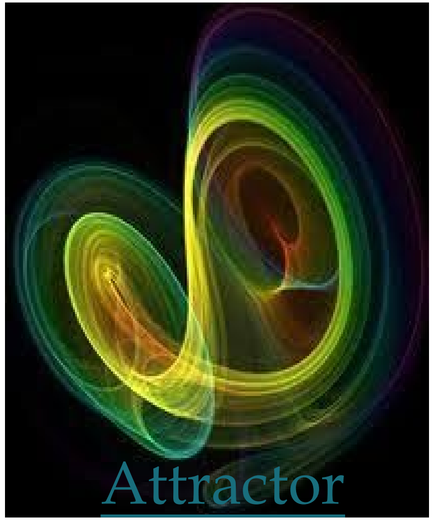


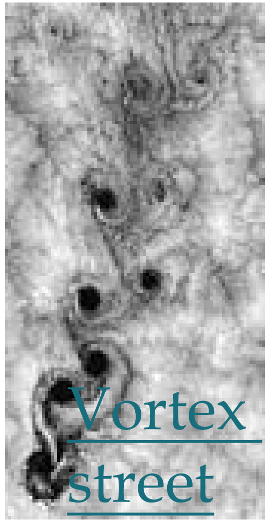
My experience of “flow patterns”

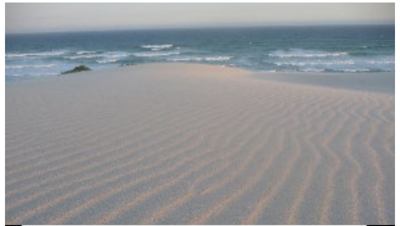
“Flow instability, modelling and control”

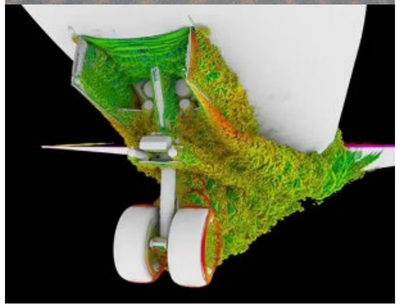


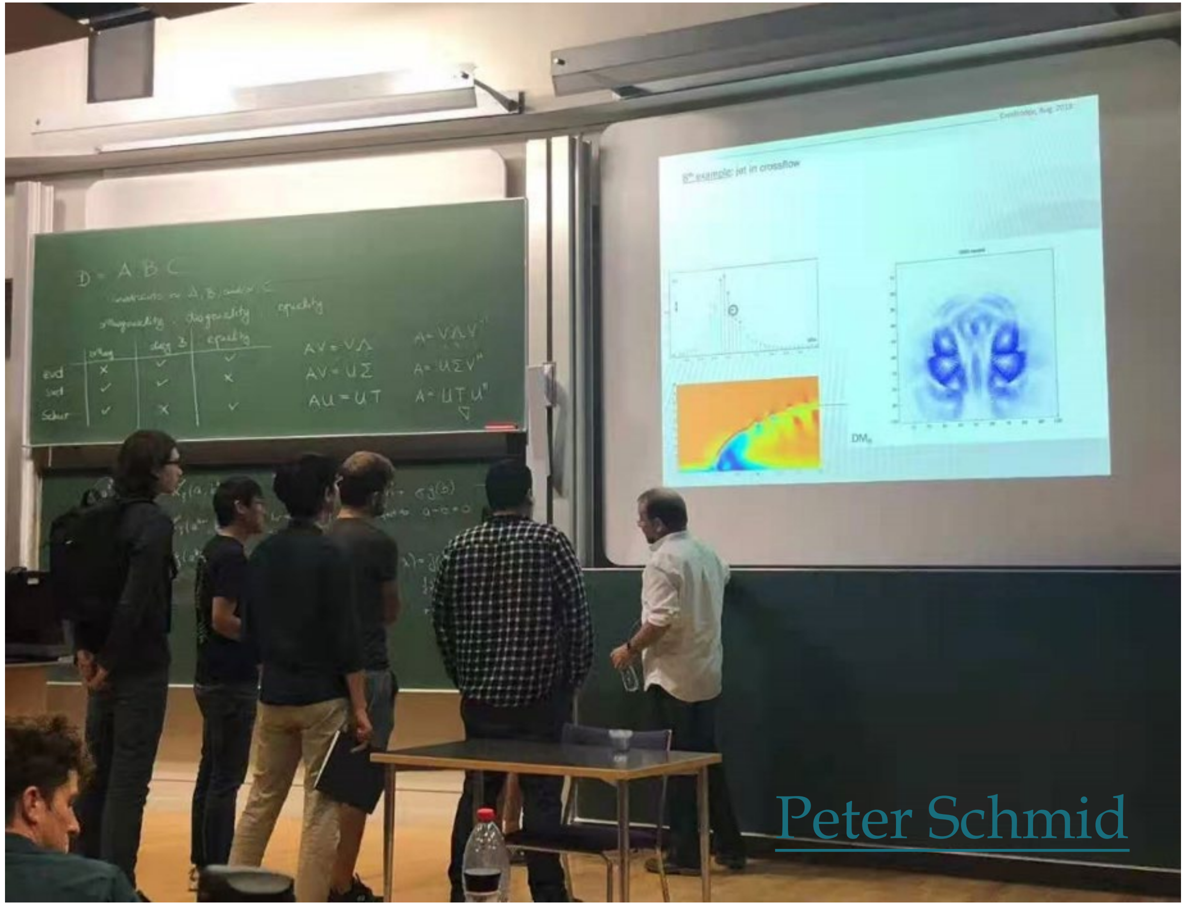
https://fluids.ac.uk/sig/FlowInstability











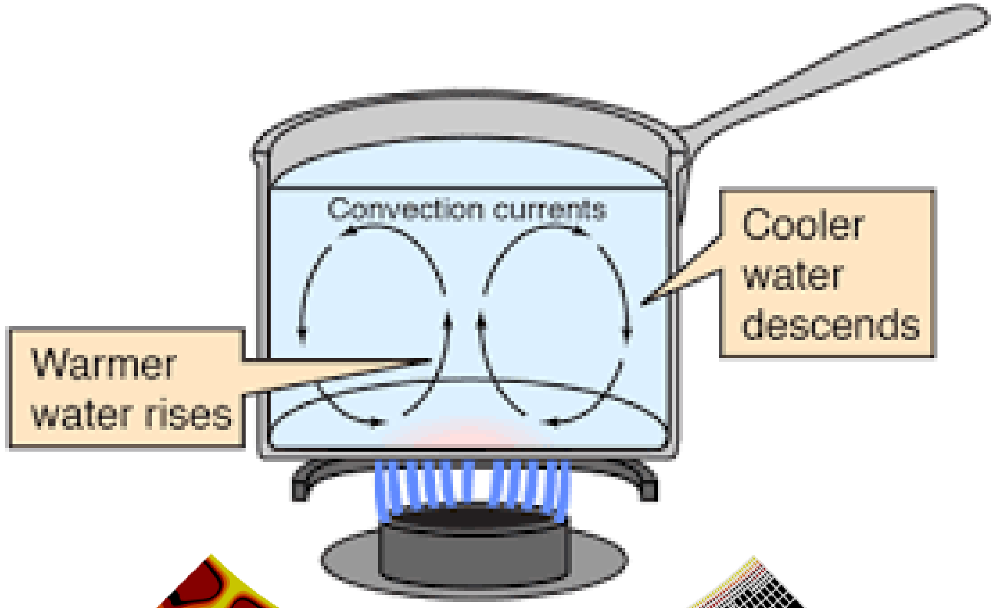
At same time, another workshop quietly launched in Cambridge

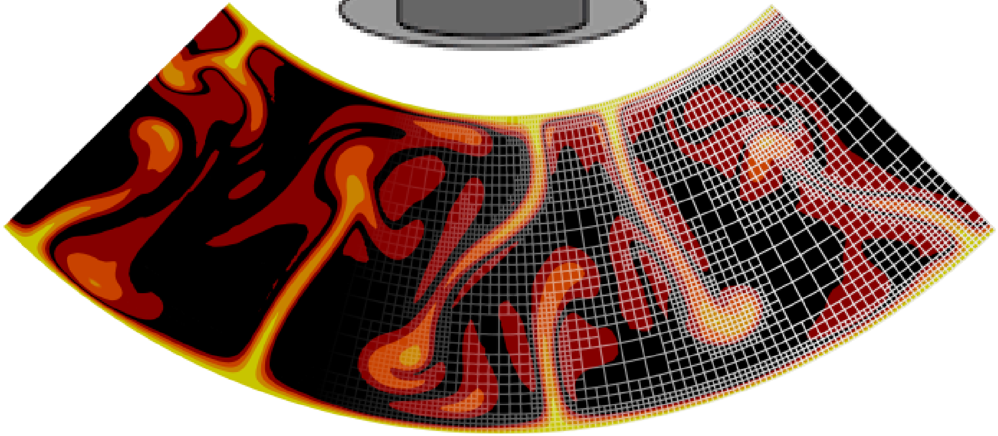
So, what is pattern?

*-Wikipedia: A pattern is a **regularity** in the world, in human-made design, or in abstract ideas. As such, the elements of a pattern **repeat in a predictable** manner.*

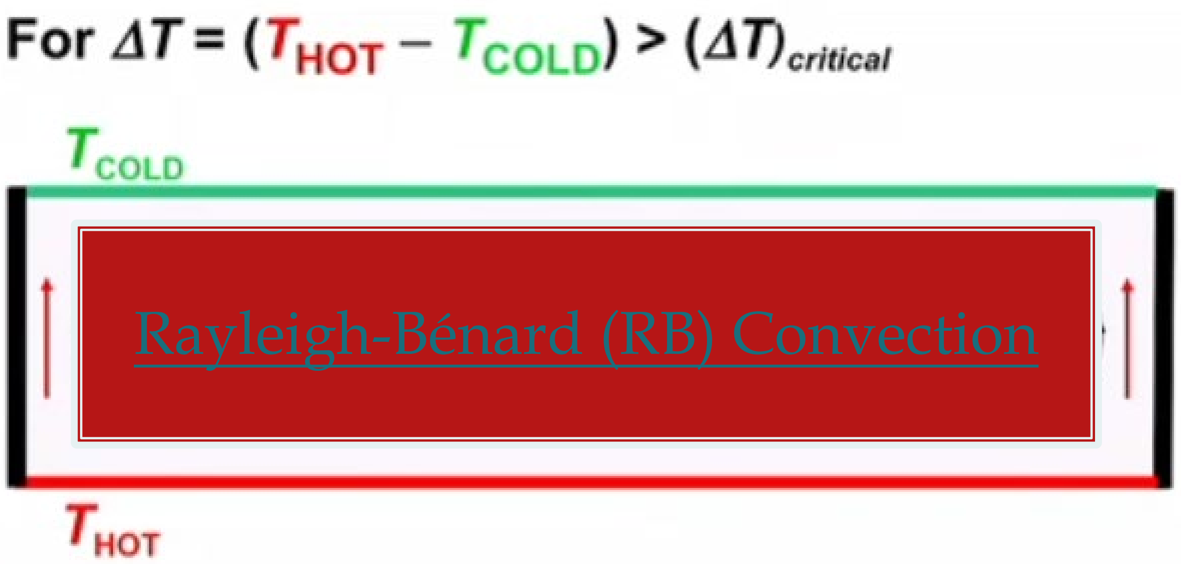
- In chao's theory: $\frac{d}{dt}\mathbf{x}(t) = \mathbf{f}(\mathbf{x}(t), t; \boldsymbol{\beta})$ → Koopman operator $g(\mathbf{x}_{k+1}) = \mathcal{K}_t g(\mathbf{x}_k).$
- In fluid mechanism: $\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u} - \boxed{\nu\nabla^2\mathbf{u}} = -\nabla w + \mathbf{g}$ Nonlinear — Naiver-stoke equation
- In mathematician's eyes : $A\vec{x} = \lambda\vec{x}$ — $\vec{x}$ = eigenvector, λ = eigenvalue
- In engineer's eyes : Resonance / Mode

System in thermodynamic equilibrium

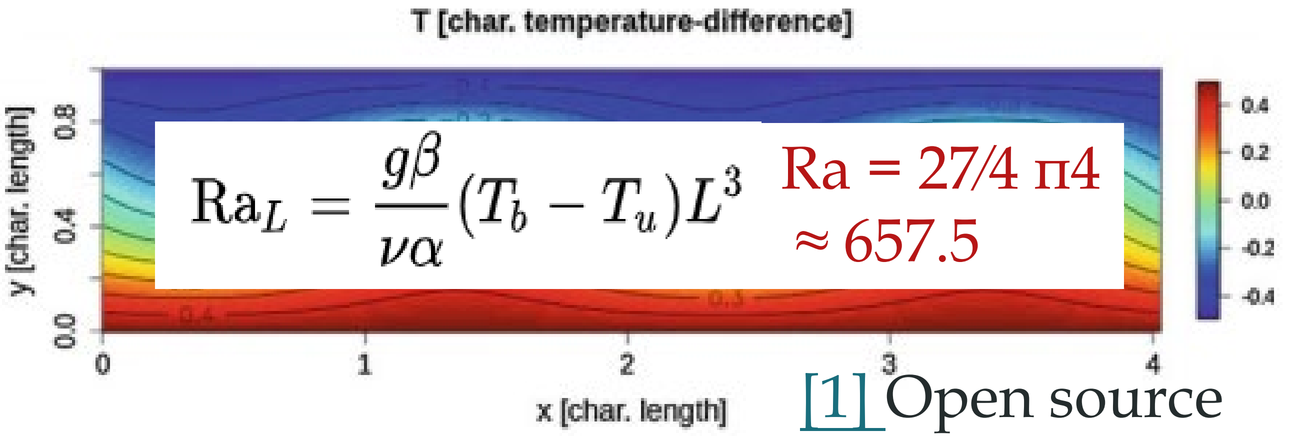




Aspect solver for Earth's Convectic



Rayleigh-Bénard (RB) Convection



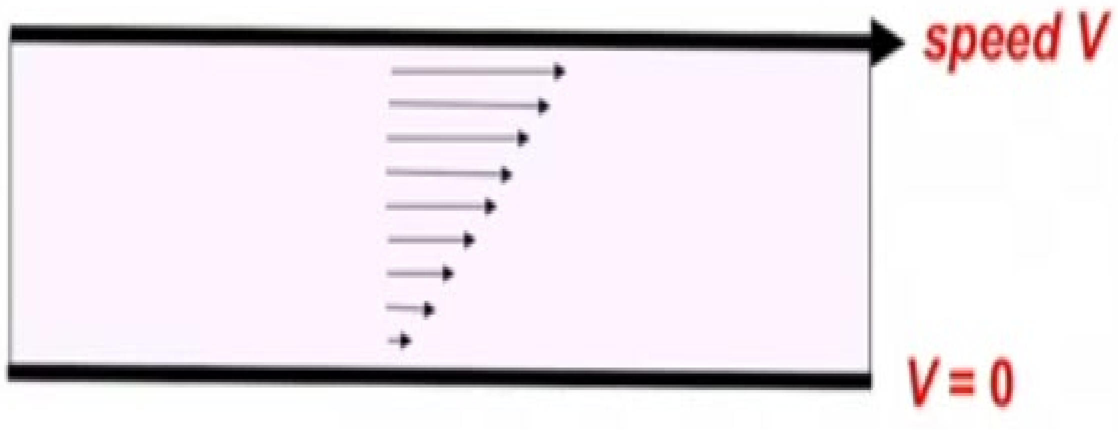
$$\mathrm{Ra}_L = \frac{g\beta}{\nu\alpha}(T_b - T_u)L^3$$

$\mathrm{Ra} = 27/4\,\pi^4 \approx 657.5$

[1] Open source

Is all the emergence of a pattern predictable?

- Consider a fluid between two parallel plates:

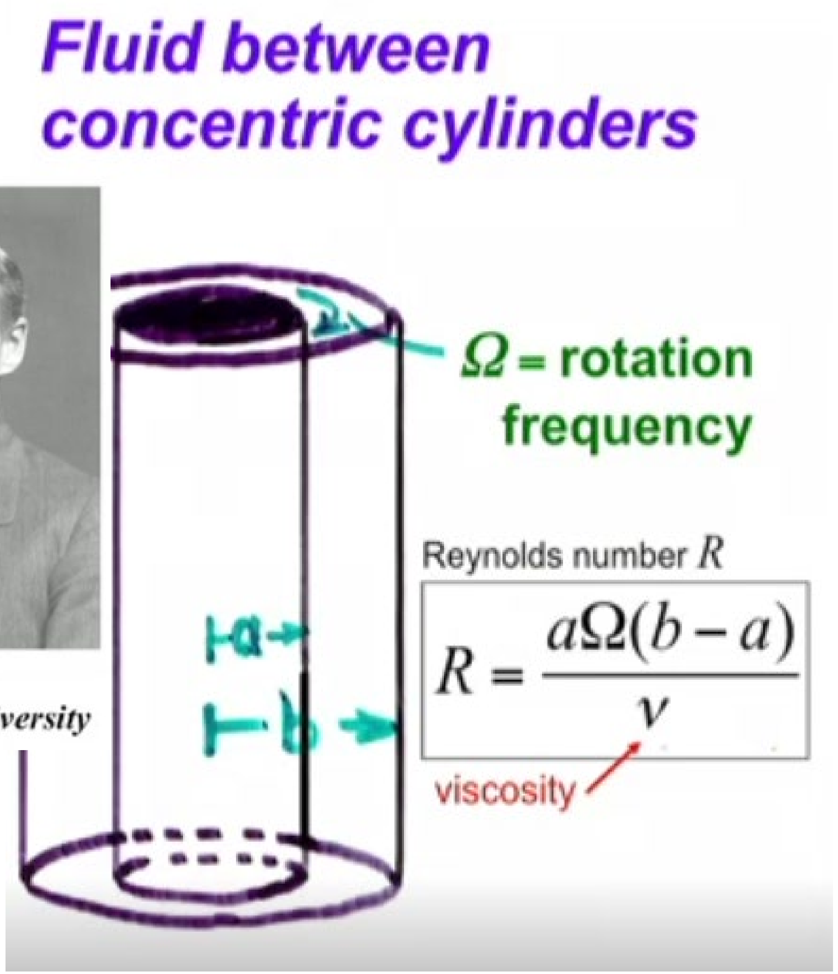


How large does V have to be for the flow to change the pattern?

Early 20th century:

All attempts to predict the answer failed.

G.I. Taylor
Cambridge University



Is all the emergence of a pattern predictable?

Emergence of a pattern: $R > R_c$ Emergence of Taylor vortex pattern

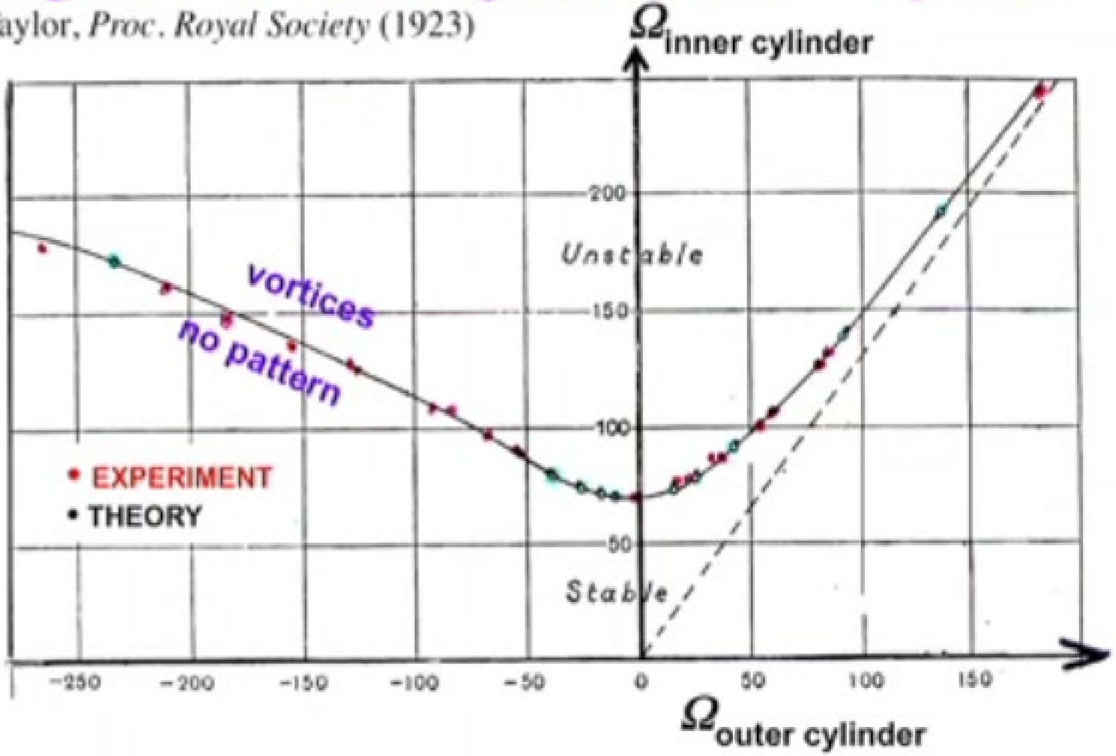
←donut-shaped vortices

vortex pair

G. I. Taylor
Phil. Trans. Roy. Soc. (1923)

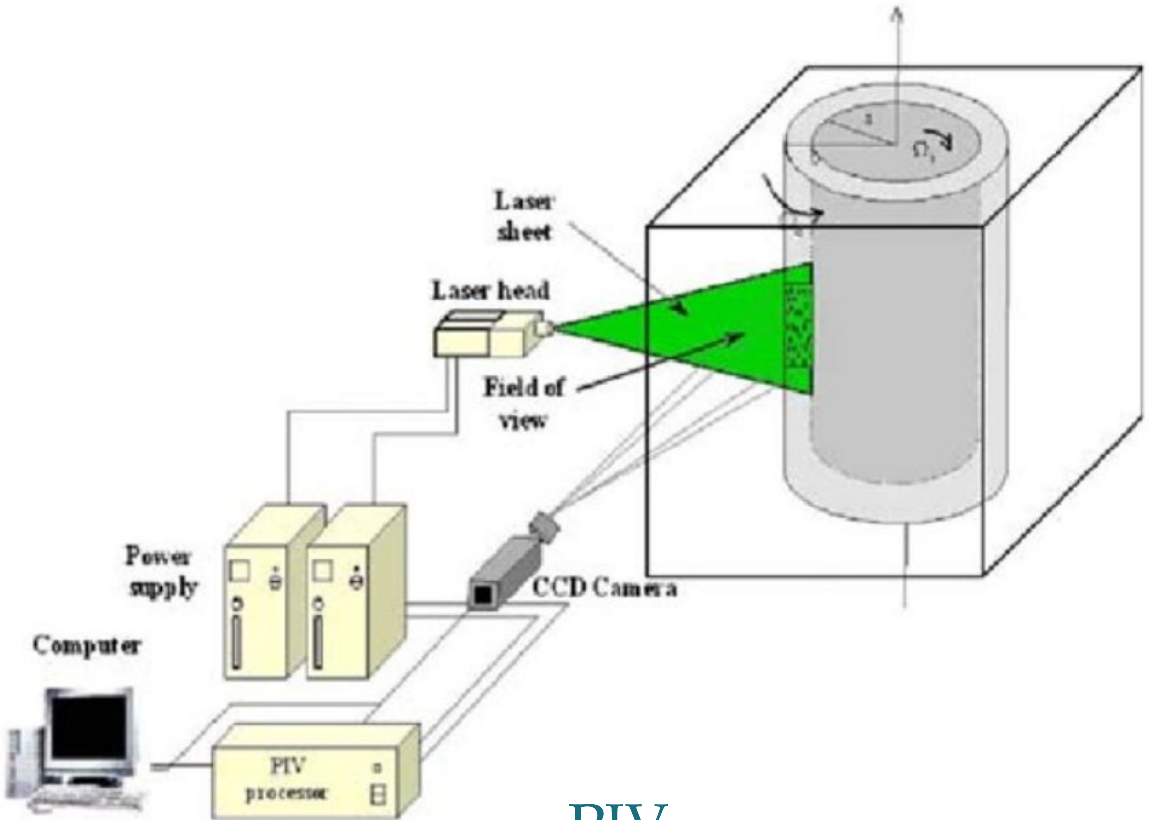
Photo: Andereck, Liu, Swinney
J Fluid Mech. (1985)

G. I. Taylor, *Proc. Royal Society* (1923)

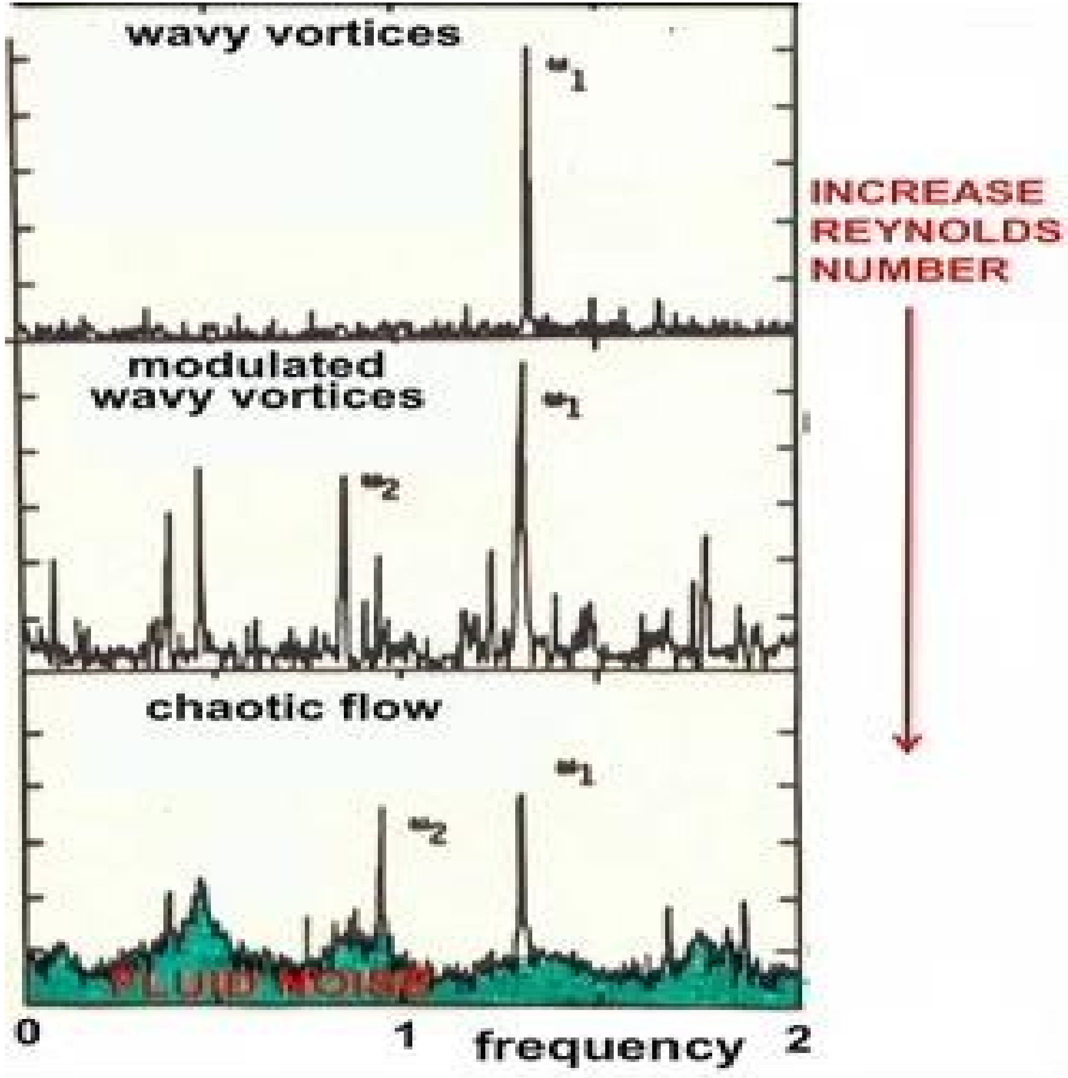


Attempts by Kelvin, Rayleigh, Hopf, Sommerfeld, and others
"to calculate the speed at which any type of flow would become unstable have failed."

Experiment Taylor–Couette flow system

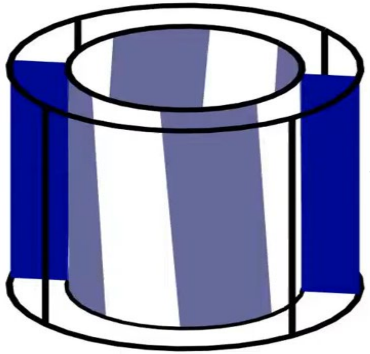


PIV



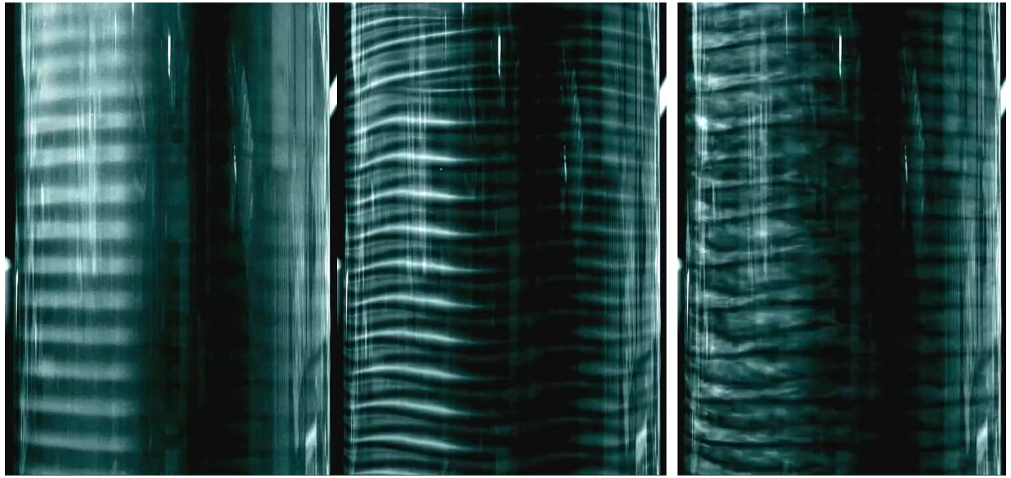
Explored Taylor–Couette flow system

$$Ta = \frac{\Omega^2 R_1 (R_2 - R_1)^3}{\nu^2}$$, **Ta = 0**



Simulation by FEATool Multiphysics

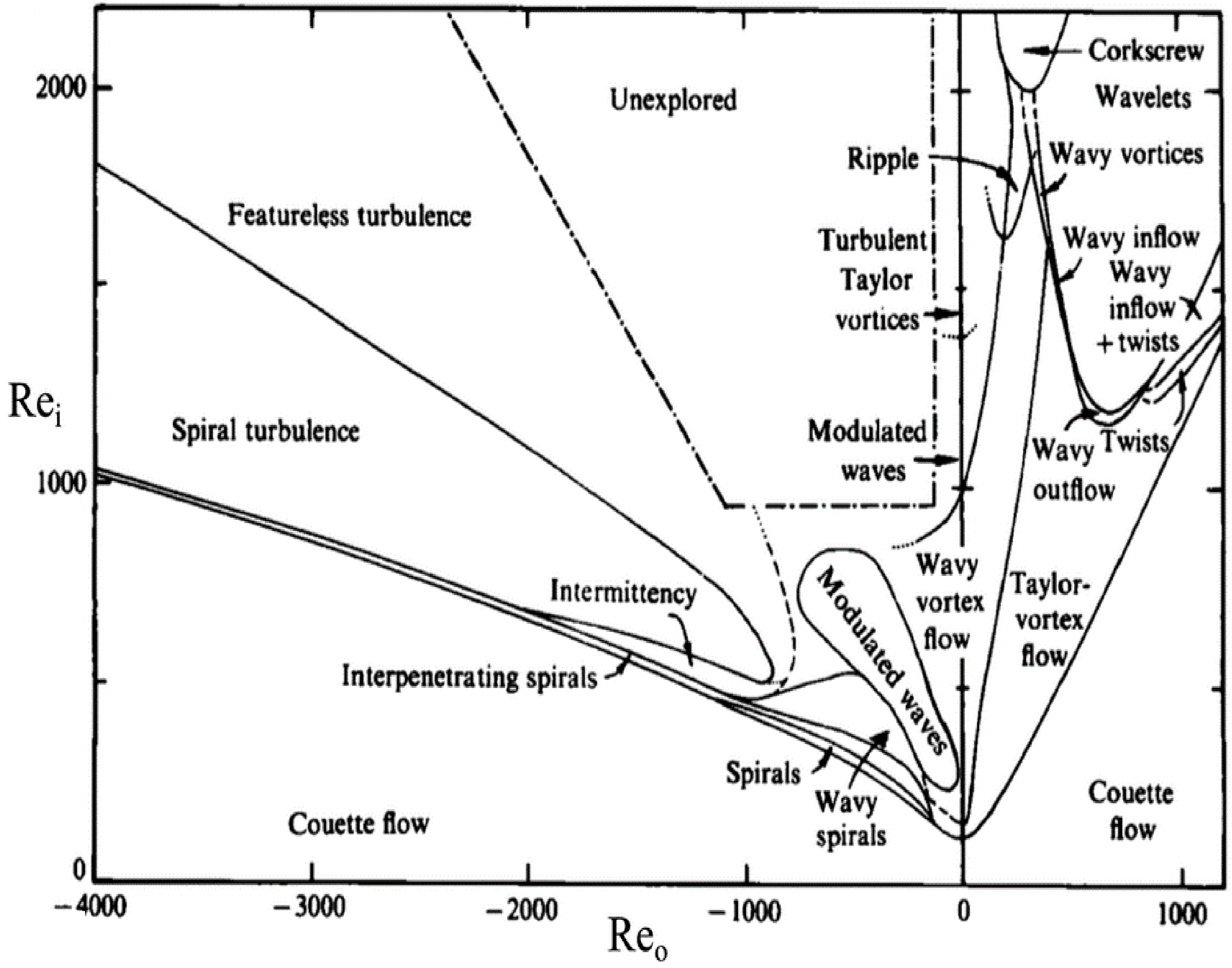
Increase Reynolds number



Taylor vortices wavy vortices chaotic flow

[2]:Ruy Ibanez, Phys. Rev. Fluids,2016

[3]: M. A. Fardin-2014

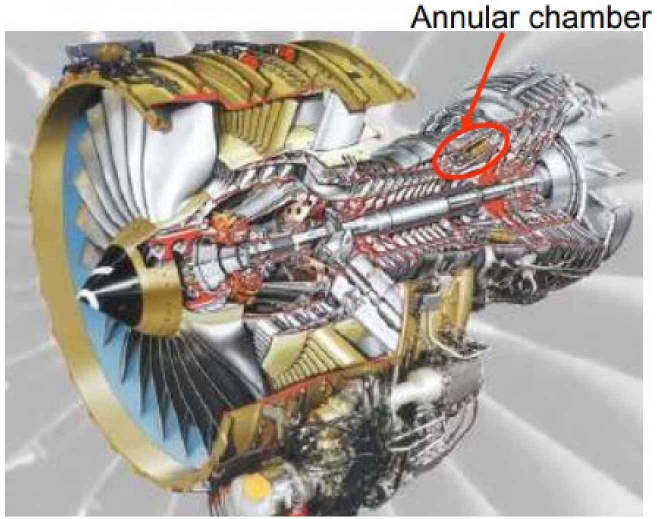


If patterns exit in more complex fluid dynamical system,
Such like turbomachinery?
Well, yes, some could be simplify,
but more others are headache, confusing!

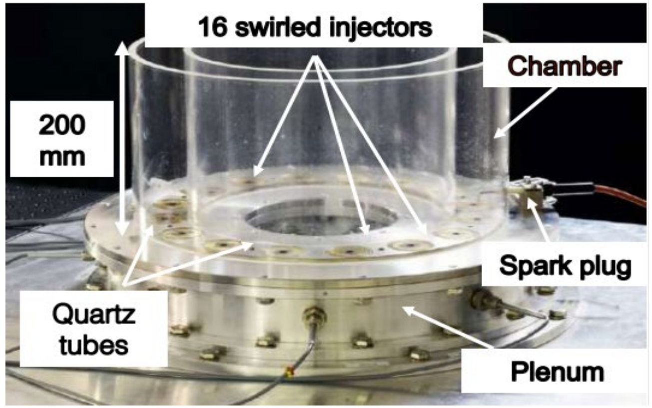
Contact me:
www.deal-ii.com

Combustion instability in ducts

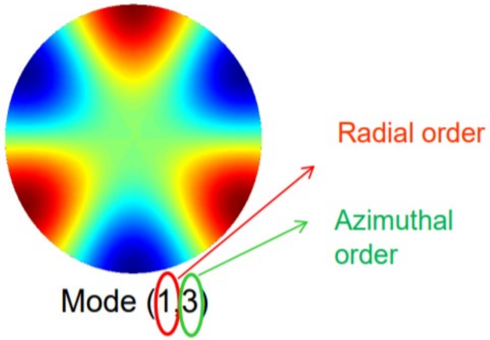
CFM 56 turbofan



Annular combustor MICCA-Spray



Harmonic modes are governed by a Helmholtz equation



$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 p}{\partial \theta^2}+\frac{\partial^2 p}{\partial z^2}=\frac{1}{c^2}\frac{\partial^2 (rp)}{\partial t^2}$$

Assuming that the wall is rigid, the normal velocity of the pipe wall is 0

$$u_{rm}\Big|_{r=a}=0 \Rightarrow \left.\frac{dJ_m(k_{mn}r)}{d(k_{mn}r)}\right|_{r=a}=0$$

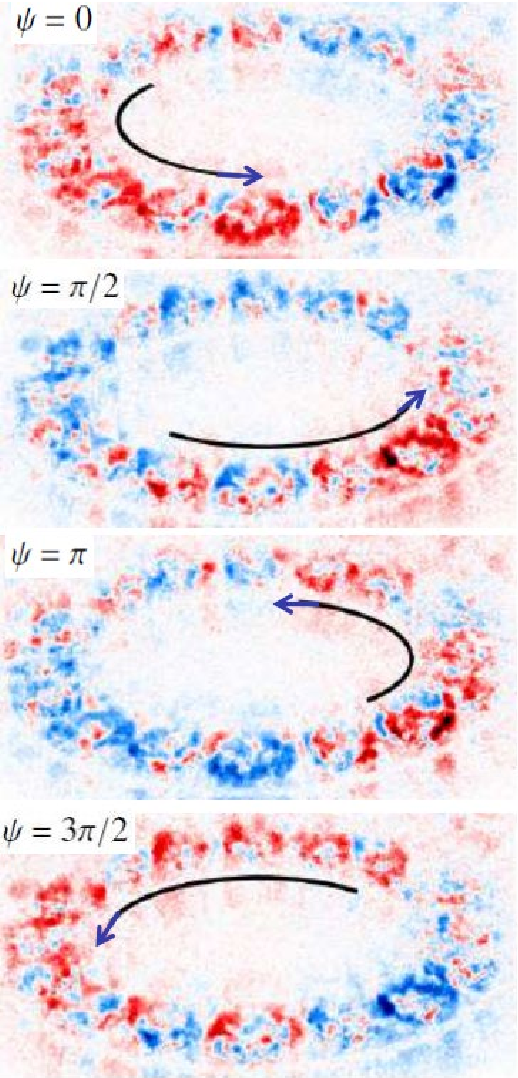
➢ pressure mode superposition：

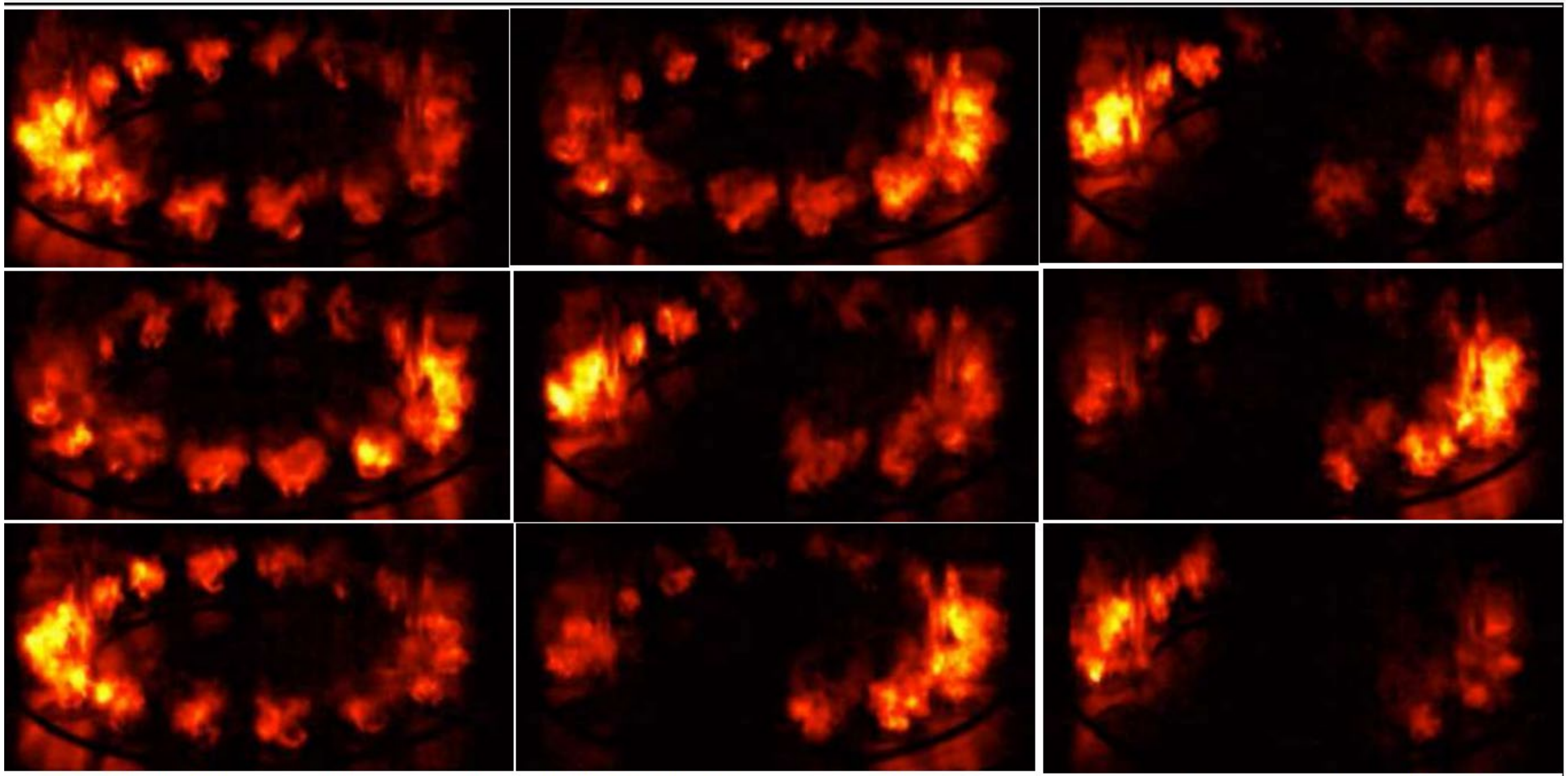
$$p=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}A_{mn}J_m(k_{mn}r)\cos(m\theta-\varphi_m)e^{j(\omega t-k_z z)}$$

$$\text{轴向波数}k_z=\sqrt{k^2-k_{mn}^2},\ k^2=\frac{\omega^2}{c^2}$$

Axial wave number

Combustion rotating modes captured



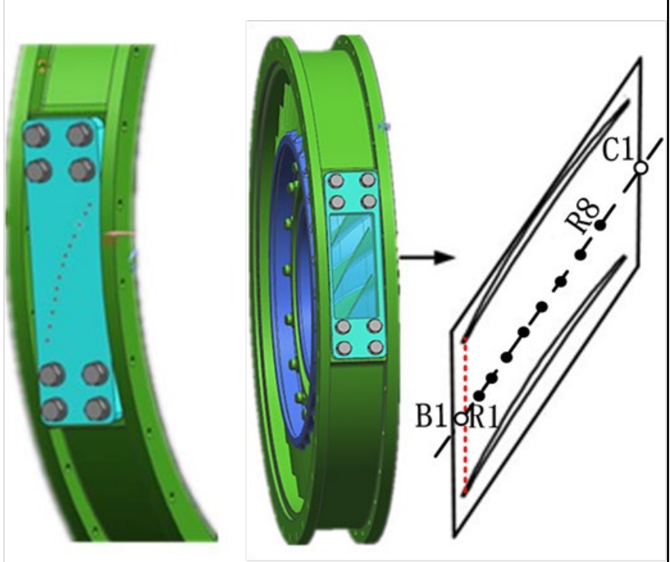


Partial flame blow off in MICCA-Spray in the presence of an azimuthal standing mode of high amplitude (4000 Pa peak)

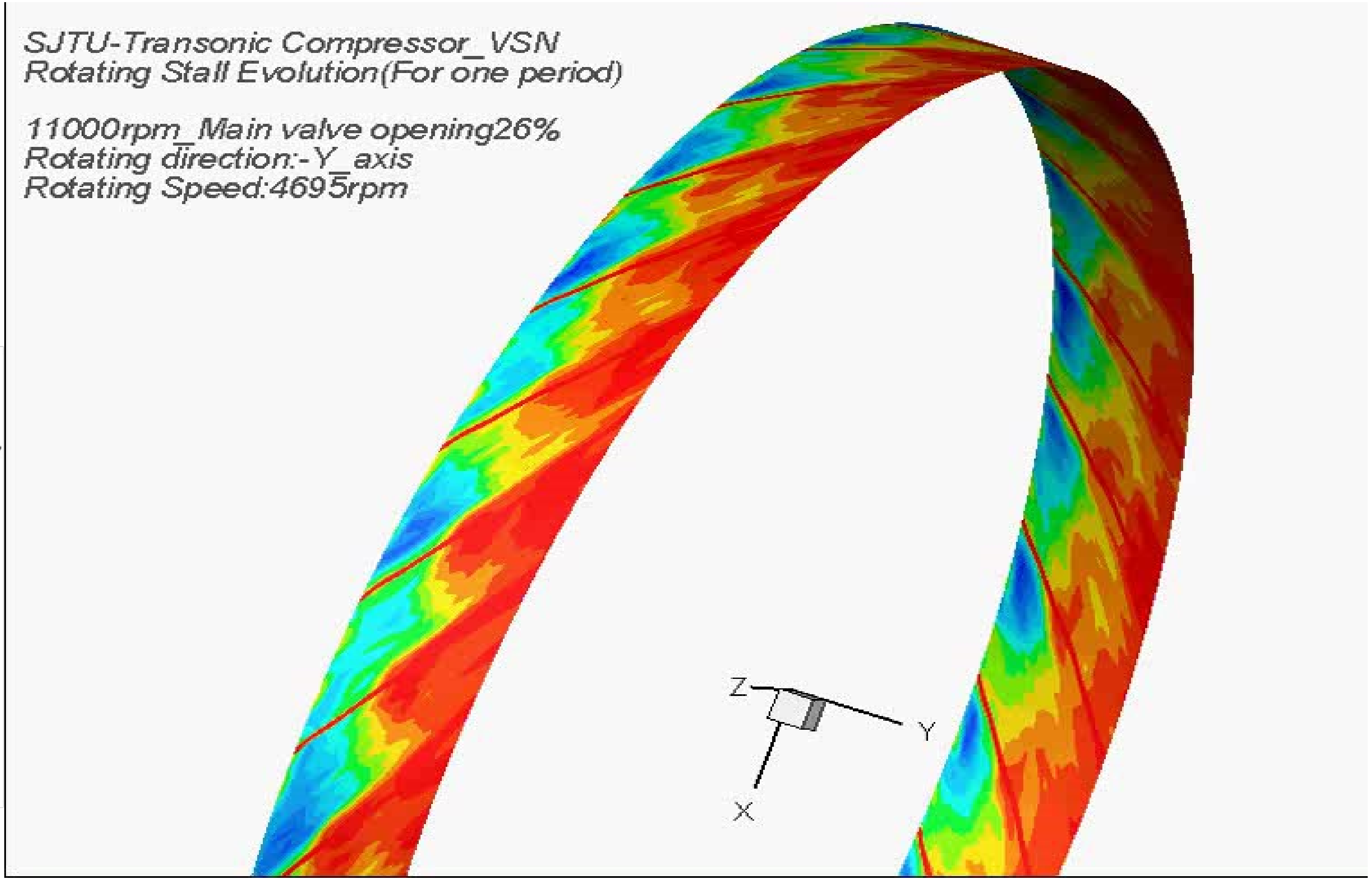
[4]:S. Candel- Princeton summer school, June 2019

Compressor rotating stall

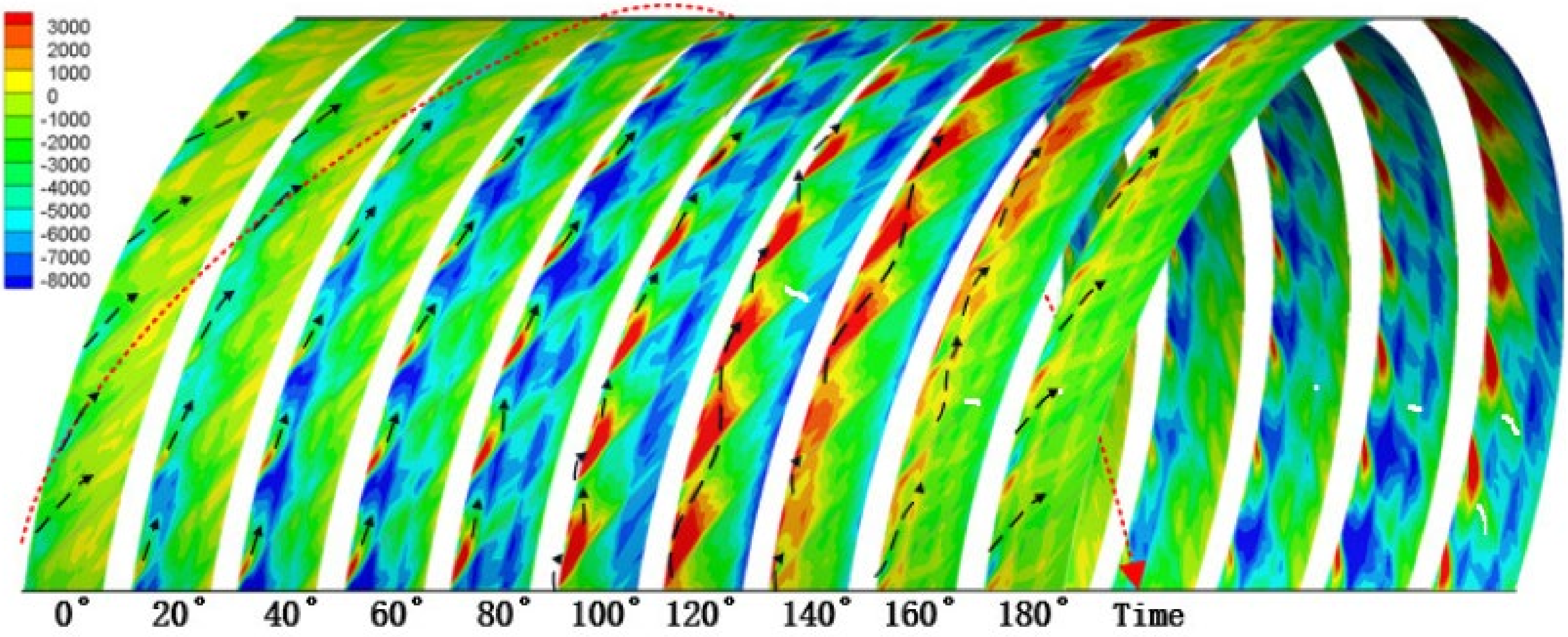
KuliteXCL-062



chord measurement layout of rotor



Pattern of rotating stall

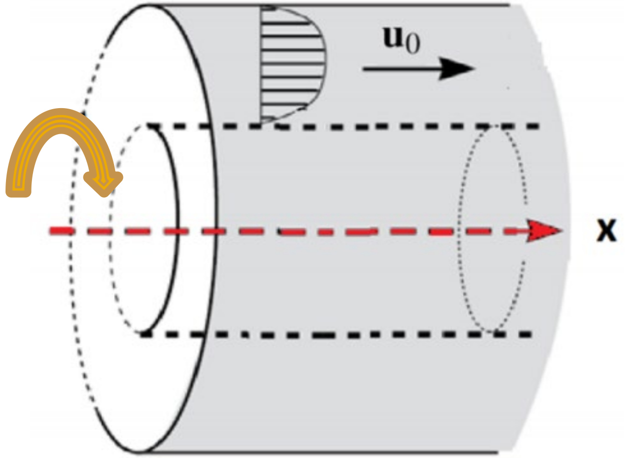


Wave pattern in swirling flow

- Should be considered both for combustion and aeroacoustics

- Factor 1： $R1 \le r \le R2$
- Factor 2： $\mathrm{u}_0(r)$
- Factor 3： $\mathrm{u}_\theta(r)$
- Factor 4： Z_t, Z_h
- Factor 5： $\mathrm{s}_0(r)$
- Factor 6： $y_1(x) \le r(x) \le y_2(x)$
- Factor 7： Conical Duct
- Factor 8： viscosity、turbulence

Realistic flow



Realistic flow: Eigenvalue

$$\{u,\ v,\ w,\ p,\ s\}(r,\ x,\ \theta,\ t)=\int\sum_{\mathrm{n}}\int\{U(r),V(r),W(r),P(r),S(r)\}e^{ikx}dke^{in\theta}e^{-i\omega t}d\omega$$

Linear Euler equations :

$$\begin{cases}
\dfrac{1}{c_0^{\,2}}\dfrac{D_0p}{Dt}+\dfrac{\rho_0U_\theta^{\,2}}{rc_0^{\,2}}v+\rho_0(\nabla\cdot u)=0\\[2mm]
\rho_0(\dfrac{D_0u}{Dt}+v\dfrac{dUx}{dr})+\dfrac{\partial p}{\partial x}=0\\[2mm]
\rho_0(\dfrac{D_0v}{Dt}-\dfrac{2U_\theta w}{r})-\dfrac{U_\theta^{\,2}}{r}\rho+\dfrac{\partial p}{\partial r}=0\\[2mm]
\rho_0\left(\dfrac{D_0w}{Dt}+\dfrac{v}{r}\dfrac{d}{dr}(rU_\theta)\right)+\dfrac{1}{r}\dfrac{\partial p}{\partial\theta}=0\\[2mm]
\dfrac{D_0s}{Dt}+\dfrac{ds_0}{dr}v=0
\end{cases}$$

$$\Omega(r)=\omega-kU_x(r)-\frac{nU_\theta(r)}{r}$$

$$\hat{\Omega}=\omega-\frac{nU_\theta}{r},\zeta=1-U^2{}_xc^2{}_0$$

Wave domain equation

$$A=\begin{bmatrix}
\dfrac{U_x\hat{\Omega}}{c^2{}_0\zeta}i & \left[-\dfrac{U_x}{c^2{}_0\zeta}\dfrac{dUx}{dr}+\dfrac{1}{r\zeta}+\dfrac{U_\theta^{\,2}}{\zeta rc^2{}_0}\right]+\dfrac{1}{\zeta}\dfrac{d}{dr} & \dfrac{m}{r\zeta}i & -i\dfrac{\hat{\Omega}}{c^2{}_0\rho_0\zeta} & 0\\[2mm]
0 & -i\dfrac{\hat{\Omega}}{U_x} & -\dfrac{2U_\theta}{rU_x} & \dfrac{1}{\rho_0U_x}\dfrac{d}{dr}-\dfrac{U_\theta^{\,2}}{\rho_0U_xrc^2{}_0} & \dfrac{U_\theta^{\,2}}{rc_pU_x}\\[2mm]
0 & \dfrac{1}{U_x}\left[\dfrac{U_\theta}{r}+\dfrac{dU_\theta}{dr}\right] & -\dfrac{\hat{\Omega}}{U_x}i & \dfrac{im}{r\rho_0U_x} & 0\\[2mm]
-\dfrac{\rho_0\hat{\Omega}}{\zeta} & \dfrac{\rho_0}{\zeta}\left[\dfrac{dU_x}{dr}-\left(\dfrac{U_\theta^{\,2}}{c^2{}_0}+1\right)\dfrac{U_x}{r}\right]-\dfrac{\rho_0U_x}{\zeta}\dfrac{d}{dr} & -\dfrac{m\rho_0U_x}{r\zeta} & i\dfrac{U_x\hat{\Omega}}{c^2{}_0\zeta} & 0\\[2mm]
0 & \dfrac{1}{U_x}\dfrac{ds_0}{dr} & 0 & 0 & -i\hat{\Omega}
\end{bmatrix}$$

$$\mathbf{X}=\begin{bmatrix}U\\V\\W\\P\\S\end{bmatrix}$$

$$\boxed{\lambda=-ki}$$

Numerical method

Chebyshev Point

$$r_j = \cos\frac{\pi j}{N},\ j = 0,\cdots,\mathrm{N}$$

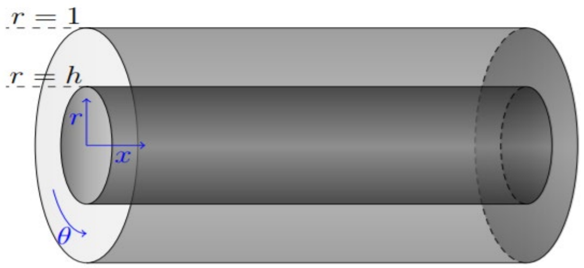
$$f(r) = \sum_{j=0}^{N} f(r_j)\, g_j(r)$$

$$\frac{df(r_i)}{dr} = \sum_{j=0}^{N} D_{ij} f(r_j),\ i = 0,\cdots,N$$

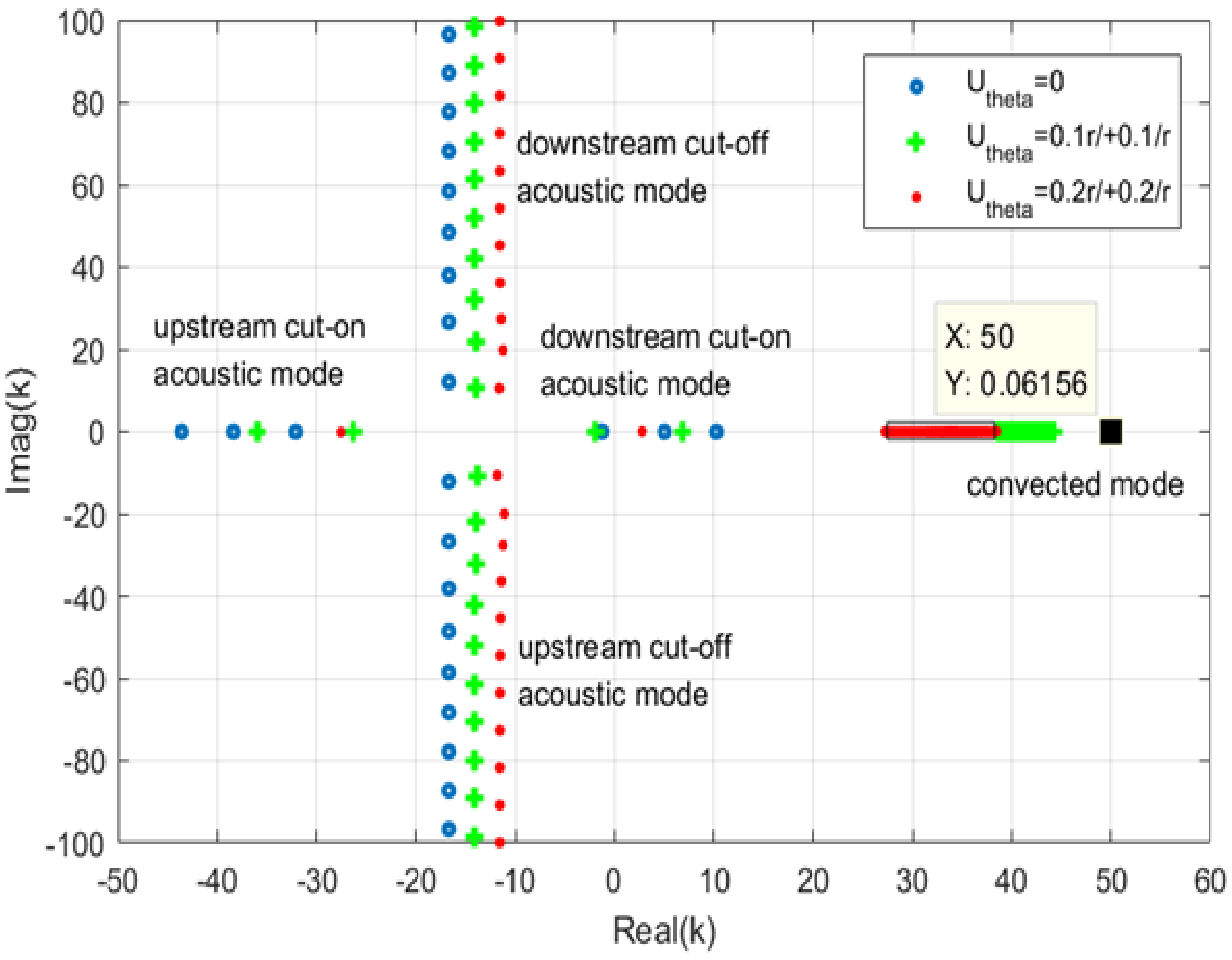
$$D_{ij} = \begin{cases} \dfrac{c_i(-1)^{i+j}}{2c_j \sin\frac{\pi}{2N}(i+j)\sin\frac{\pi}{2N}(-i+j)} & i \neq j; i = 0,\cdots N/2;\ j = 0,\cdots,N \\ \dfrac{-\cos(\frac{\pi i}{N})}{2\sin^2(\frac{\pi i}{N})} & i = j; i = 1,\cdots N/2;\ j = 1,\cdots,N \\ \dfrac{2N^2+1}{6} & i = j = 0 \\ -D_{N-i,N-j} & i = N/2+1,\cdots N;\ j = 0,\cdots,N \end{cases}$$

Ingard-Myers boundary condition:

$$\begin{cases} Z_h \dfrac{\omega V(h)}{U_x(h)} + \dfrac{\hat{\Omega}(h)P(h)}{U_x(h)} - kP(h) = 0 \\ Z_1 \dfrac{\omega V(1)}{U_x(1)} - \dfrac{\hat{\Omega}(1)P(1)}{U_x(1)} + kP(1) = 0 \end{cases}$$



Explore the pattern from eigenvalue



h=0. 6, w=25, n=15, Ux=0. 5 solid wall

- The eigenvalue is the axial wave number, which reflects the acoustic propagation characteristics of the pipe.
- (near/pure) acoustic mode
- Vortex-dominant (near/pure) convective mode
- Swirling changes the cutoff and propagation characteristics of pipe acoustic propagation
- Swirl is one of the acoustic resonance factors in the area of the inlet chute (Copper&Peake 2010 JFM)

$$\{p\}(r, x, \theta, t) = \int \sum_{n} \int \{P(r)\} e^{ikx} dk e^{in\theta} e^{-i\omega t} d\omega$$

- *no swirl* cut-off line

$$real(k) = -wU_x / \left(1 - U_x^2\right)$$

- convected mode

$$-w + kU_x + mU_\theta / r$$

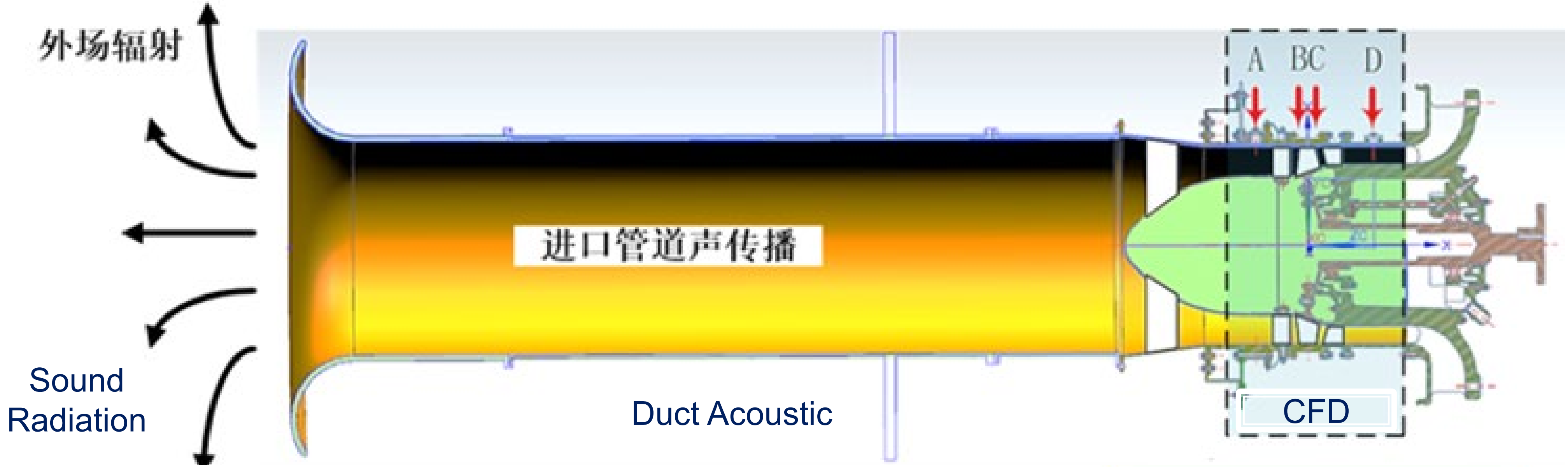
no swirl

$$\inf_{h<r<1} \frac{w}{U_x(r)} \leq k \leq \sup_{h<r<1} \frac{w}{U_x(r)}$$

swirl

$$\inf_{h<r<1} \frac{w\text{-}nU_\theta(r)}{U_x(r)} \leq k \leq \sup_{h<r<1} \frac{w\text{-}nU_\theta(r)}{U_x(r)}$$

Application: Acoustic Analogy



Constructive interference: sound of the total fan = **B** x (sound of a single blade)

Bessel function: modulation of the Doppler frequency shift during blade revolution

Tonal Blade Noise Modeling

$$p_{nB} \sim \frac{iBk_{nB}}{4\pi x}\, e^{ik_{nB}x} \sum_{p=-\infty}^{+\infty} e^{i(nB-p)(\varphi-\pi/2)} J_{nB-p}(k_{nB}R_0 \sin\theta) \left[\overline{F}_{T,p}\cos\theta - \frac{nB-p}{k_{nB}R_0}\overline{F}_{D,p}\right]$$

Sound emitted at **BPFHs**

Sum over **BLHs**

Radius where force is applied

Thrust harmonic

Drag harmonic

Acoustic Analogy: realistic flow

Lighthill eq.:

Quadrupole　　Dipole　　Monopole

$$\rho'(\vec{x},t)=\frac{1}{c_0^{\ 2}}\int_{-T}^{T}\int_{v(\tau)}\frac{\partial^2 G}{\partial y_i \partial y_j}T'_{ij}d\vec{y}d\tau+\frac{1}{c_0^{\ 2}}\int_{-T}^{T}\int_{S(\tau)}\frac{\partial G}{\partial y_i}f_i dS(\vec{y})d\tau+\frac{1}{c_0^{\ 2}}\int_{-T}^{T}\int_{S(\tau)}\rho_0 V'n\frac{D_0 G}{\partial \tau}dS(\vec{y})d\tau$$

Through the eigenfunction method, the Green's formula for satisfying the boundary conditions of the pipe wall for an infinitely long pipe can be derived.

$$G(\vec{y},\tau\mid\vec{x},t)=\frac{i}{4\pi}\sum_{m,n}\frac{\Psi_{m,n}(y_2,y_3)\Psi^*_{m,n}(x_2,x_3)}{\Gamma_{m,n}}\times$$

$$\int_{-\infty}^{+\infty}\frac{\exp\left\{i\left[w(\tau-t)+\frac{Mk_0}{\beta^2}(y_1-x_1)+\frac{k_{n,m}}{\beta^2}\left|y_1-x_1\right|\right]\right\}}{k_{n,m}}dw$$

Posson&Peake eq.: (JFM 2012)

Considering the influence of the axial shear and the rotating base flow in the pipeline, the equation is the form of the pressure disturbance under the action of the sixth-order linear operator.

$$\boxed{F^M(p') = S^M}$$

$$F^M := \left(\frac{1}{c^2_0}\frac{\bar{D}_0^{\ 2}}{Dt^2}-\frac{\bar{\partial}^2}{\partial x^2}-\frac{1}{r^2}\frac{\bar{\partial}^2}{\partial \theta^2}\right)\Re^2+\left(\frac{1}{r}\frac{\bar{D}_0}{Dt}-U'_x\frac{\bar{\partial}}{\partial x}-\Upsilon_\theta\frac{\bar{\partial}}{\partial \theta}+\left(\frac{U_\theta^{\ 2}}{rc_0^{\ 2}}-\frac{\rho'_0}{\rho 0}\right)\frac{\bar{D}_0}{Dt}\right)\Re\mathbf{T}$$

$$+\Re\frac{\bar{D}_0}{Dt}\frac{\bar{\partial}}{\partial r}\mathbf{T}-\frac{\bar{D}_0}{Dt}\left[2U'_x\frac{\bar{\partial}}{\partial x}\frac{\bar{D}_0}{Dt}+2\left(\frac{U_\theta}{r}\right)'\frac{\bar{\partial}}{\partial \theta}\frac{\bar{D}_0}{Dt}+\Im'_\theta\right]\mathbf{T}$$

- **Condition:**
  - medium (static、uniform);
  - Not observed points located in the potential flow field;
  - M < 0.3。

- **Condtion:**
  - considering the effects of swirl, non-uniform entropy, shear flow, soft wall boundary conditions, etc.;

Posson&Peake's Green function

$$\boxed{F^{M}\left(G(\vec{x},t\,|\,\vec{x}_0,t_0)\right) = \delta(\vec{x}-\vec{x}_0)\delta(t-t_0)}$$

$$\boxed{p(\vec{x},t)=\int\left(G(\vec{x},t\,|\,\vec{x}_0,t_0)\,S^{M}(\vec{x}_0,t)\right)dx_0\,dt_0}$$

FFT， expand into cylinder coordination，

$$F^{M}\left(G_w(\vec{x}\,|\,\vec{x}_0)\,e^{-iwt}\right) = \delta(\vec{x}-\vec{x}_0)\,e^{-iwt} = \delta(x-x_0)\frac{\delta(r-r_0)}{r}\delta(\theta-\theta_0)\,e^{-iwt}$$

$$p(\vec{x},t)=\int\left(G_w(r,x,\theta\,|\,r_0,x_0,\theta_0)\,S(r_0,x_0,\theta_0)\right)dx_0\,e^{-iwt}$$

Gw FFT to Wavenumber domain:

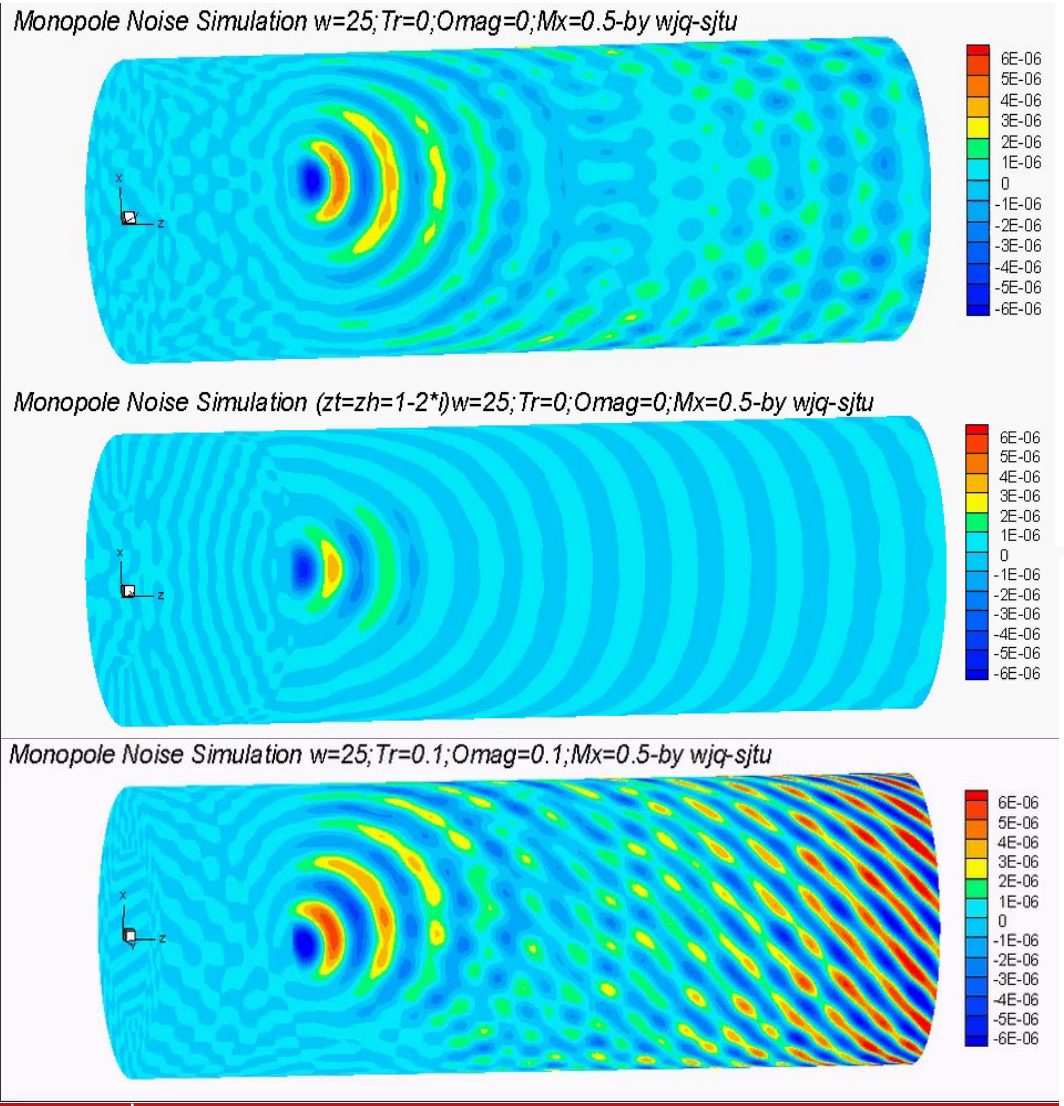
$$G_w(\vec{x}\,|\,\vec{x}_0)=\frac{1}{4\pi^2}\sum_{n=-\infty}^{\infty}e^{in(\theta-\theta_0)}\int_{i} G_n(r|r_0;w,k)\,e^{ik(x-x_0)}\,dk$$

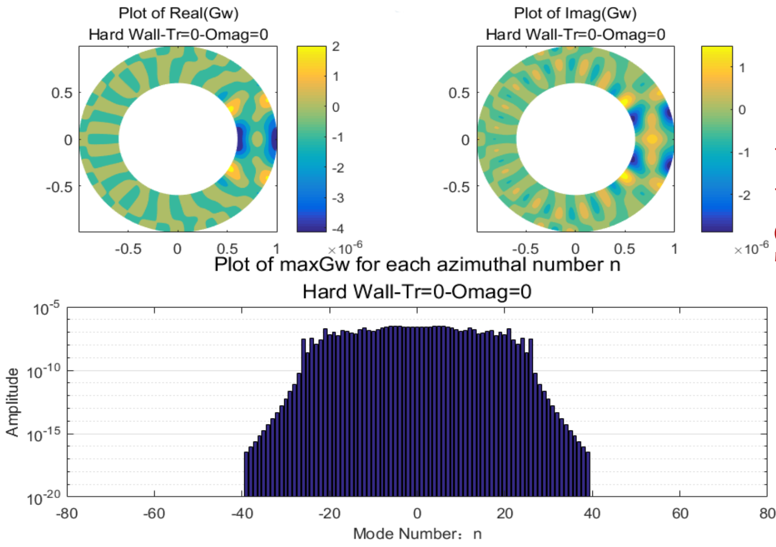
$$G_w \approx \sum_{n=-\infty}^{\infty}e^{in(\theta-\theta_0)}\sum_{Kn^{\pm}}G_n^{\ m}(x,r\,|\,x_0,r_0)$$

clue

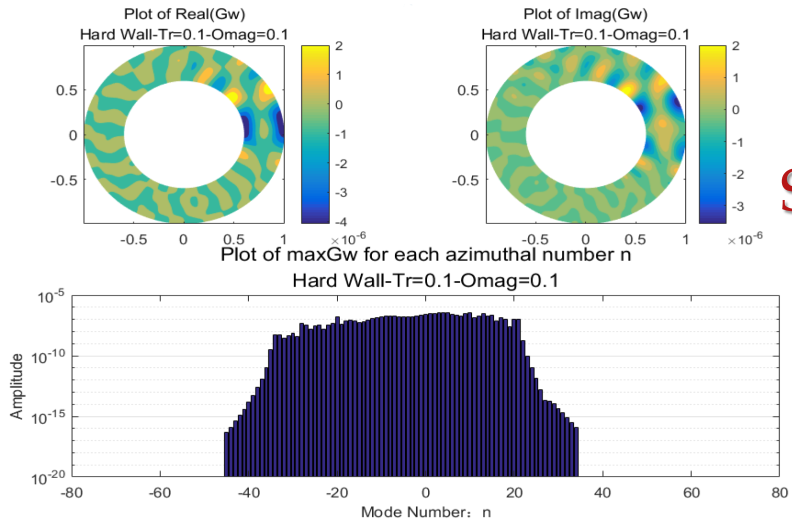
Derivation wavenumber domain Green function Gn → Satisfy B.C. $\xrightarrow{\text{eigenvalue solution}}$ Accumulate into time domain

Monopole





No Swirl

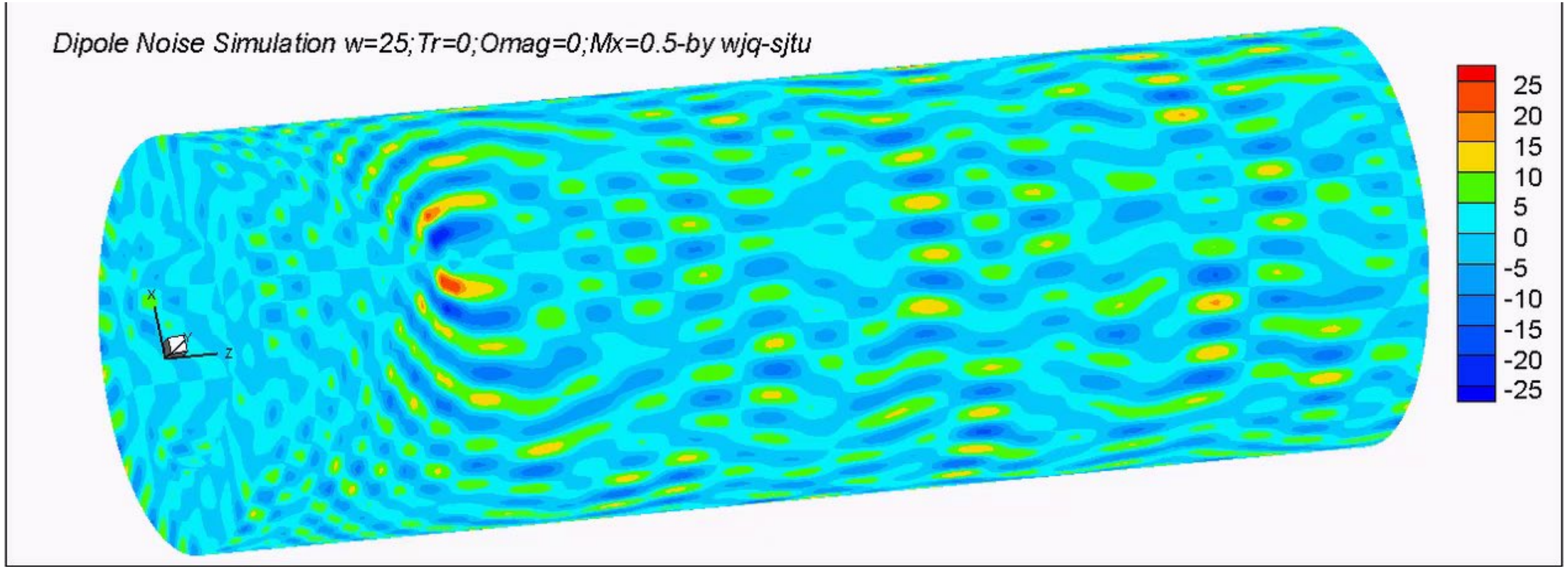


Swirl

- Sound lining regular mode and reduce noise
- The swirl changes the number of convection waves to enhance a certain order of modes

- Symmetrical distribution without swirling mode; rapid attenuation after the absolute value of the circumferential mode exceeds 25
- The swirl energy ratio is shifted to the negative direction

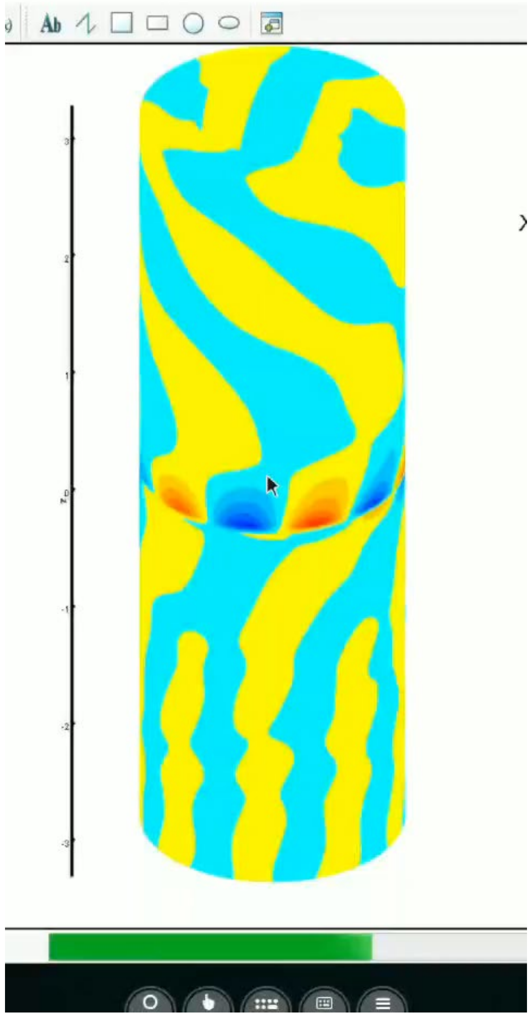
Dipole



$$p(x_d, r, \theta, t) = \iiint_{\bigcup_j \Sigma_{B,j}(t_0)} p(\mathbf{x_{d0}}, t_0) \mathcal{T}_0(G(\mathbf{x_d}, t | \mathbf{x_{d0}}, t_0)) \mathrm{d}\Sigma_0(t_0) \mathrm{d}t_0 \,,$$

$$\mathcal{T}_0(G) = \left[n_{x,j} \mathcal{D}_0^2 \frac{\partial G}{\partial x_{d0}} + n_{\theta,j} \left(\frac{\mathcal{D}_0^2}{r_0} \frac{\partial G}{\partial \theta_0} + 2 \frac{U_\theta}{r_0} \mathcal{R}_{0,1}(G) \right) \right] .$$

Rotating



$$p(x_d, r, \theta, t) = -\iint_{\cup_j S_j} \int \Delta P_j(\mathbf{x_{d0,R}}, t_0)$$

$$\times \mathcal{T}_0 \left\{ \int_\omega \sum_{m \in \mathbb{Z}} \int_k \widehat{G}_m\left(r \middle| k, \omega, r_0\right) \mathrm{e}^{\mathrm{i}\, k\, (x_d - x_{d0}) + \mathrm{i}\, m\, (\theta - \theta_{0R,j}) - \mathrm{i}\, \omega\, t + \mathrm{i}(\omega - m\Omega_R) t_0} \mathrm{d}k \mathrm{d}\omega \right\} \mathrm{d}t_0 \mathrm{d}S_{0,j} \,.$$

$$\theta_0 = \theta_{0R} + \Omega_R\, t_0$$

$$p(x_d, r, \theta, t) = 2\pi \mathrm{i} \int_\omega \sum_{m \in \mathbb{Z}} \int_k \iint_{\cup_j S_j} \Delta \hat{P}_j(\mathbf{x_{d0,R}}, \boxed{\omega_m}) \mathcal{T}_{m,k,r_0} \left(\widehat{G}_m\left(r \middle| k, \omega, r_0\right) \right)$$

$$\times\, \mathrm{e}^{\mathrm{i}\, k\, (x_d - x_{d0}) + \mathrm{i}\, m\, (\theta - \theta_{0R,j}) - \mathrm{i}\, \omega\, t} \mathrm{d}S_{0,j} \mathrm{d}k \mathrm{d}\omega \,.$$

$$\omega_m = \omega - m\Omega_R$$

Fan noise

$$\Delta P_j\left(\mathbf{x_{d0,R}}, t_0\right) = \Delta P\left(x_{d0}, r_0, \theta_{0R,j}, t_0\right) = \Delta P\left(x_{d0}, r_0, \theta_{0R,0} - \frac{2\pi}{B} j, t_0\right) = \Delta P\left(x_{d0}, r_0, \theta_{0R,0}, t_0 - \frac{2\pi}{B\Omega_R} j\right)$$
$$= \Delta P_0\left(\mathbf{x_{d0,R}}, t_0 - \frac{2\pi}{B\Omega_R} j\right),$$

$$\Delta P_j(\mathbf{x_{d0,R}}, t_0) = \sum_{q\in\mathbb{Z}} \Delta \hat{P}_{q,0}(\mathbf{x_{d0,R}})\ \mathrm{e}^{\mathrm{i}\,\frac{2\pi\, q}{B} j}\ \mathrm{e}^{-\mathrm{i}\, q\, \Omega_R\, t_0}$$

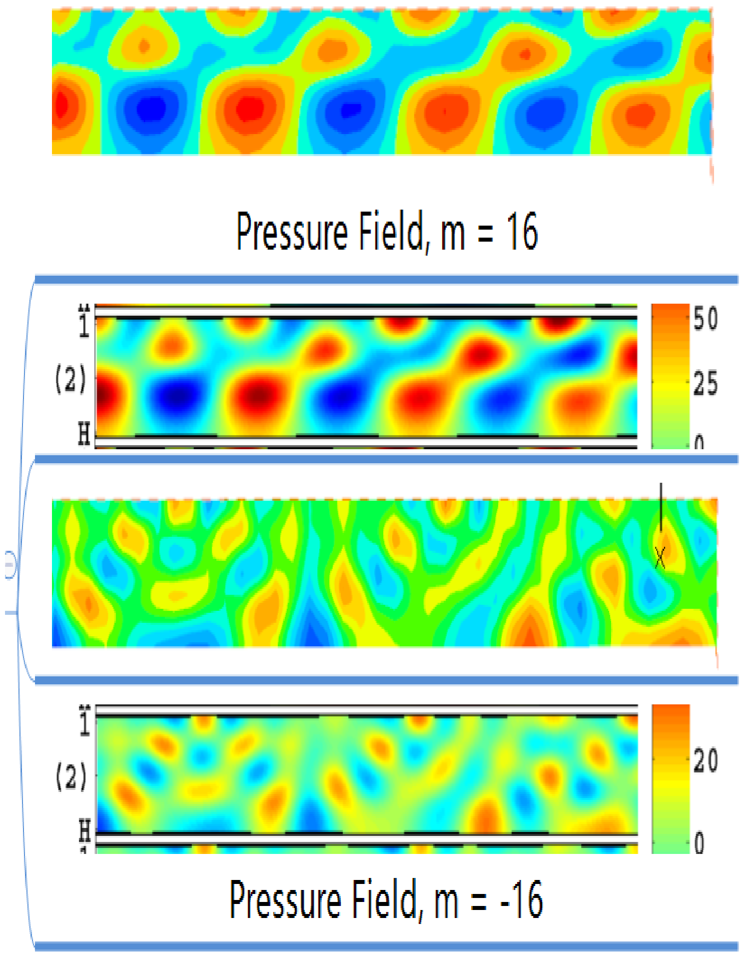
$$p(x_d, r, \theta, t) = \sum_{s\in\mathbb{Z}} \hat{p}_{sB}(x_d, r, \theta)\,\mathrm{e}^{-\mathrm{i}\, sB\Omega_R\, t} \tag{44a}$$

with

$$\hat{p}_{sB}(\mathbf{x_d}) = 2\mathrm{i}\pi\, B \sum_{q\in\mathbb{Z}} \iint_{S_0} \Delta\hat{P}_{q,0}(\mathbf{x_{d0,R}}) \int_k \mathcal{T}_{m,k,r_0}\left(\hat{G}_m\left(r\middle| k, sB\Omega_R, r_0\right)\right)\, \mathrm{e}^{\mathrm{i}\,k\,(x_d - x_{d0})}\mathrm{d}k\ \mathrm{e}^{-\mathrm{i}\,m\,\theta_{0R,0}}\mathrm{d}S_{0,0}\ \mathrm{e}^{\mathrm{i}\,m\,\theta}, \tag{44b}$$

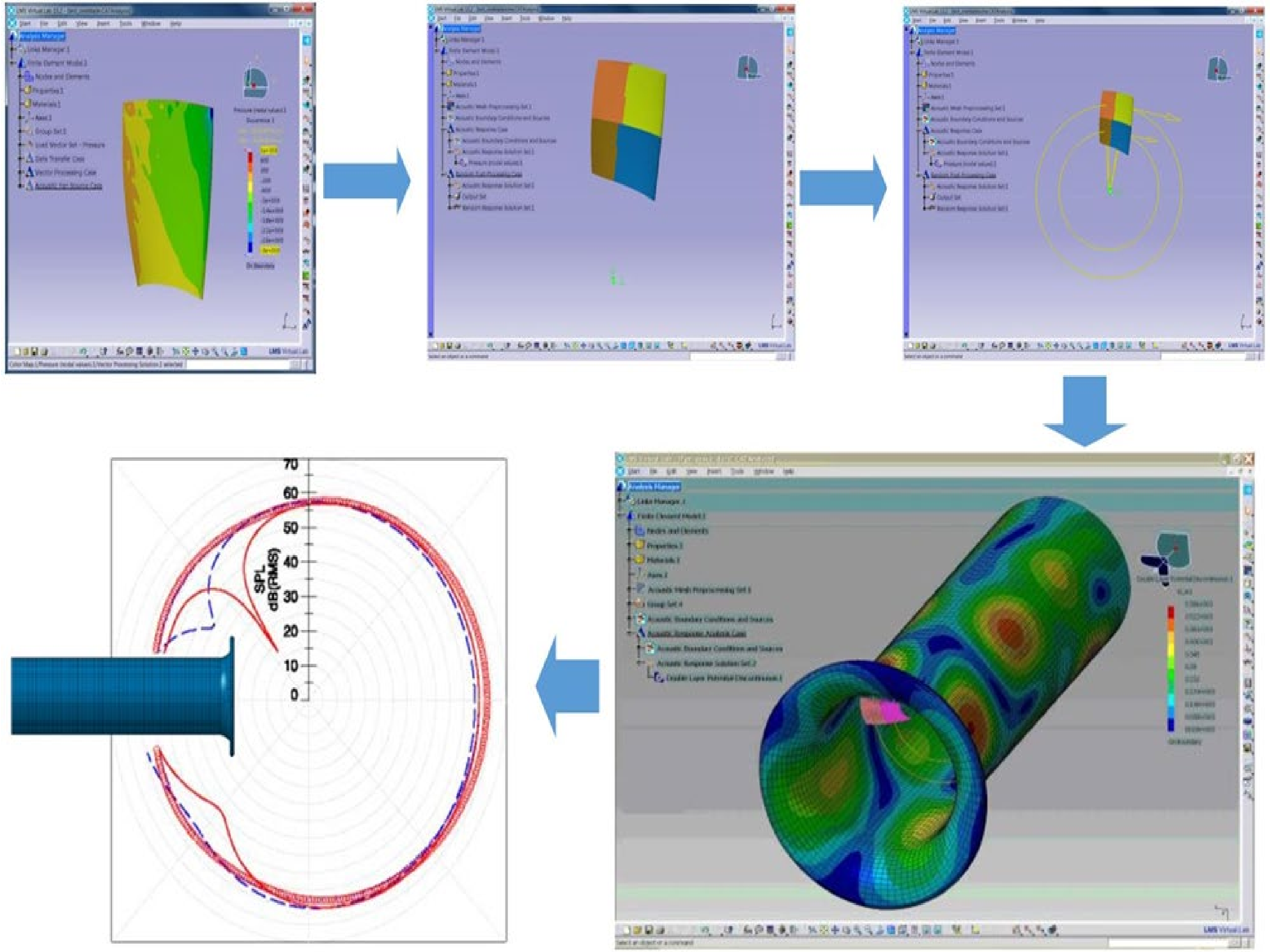
and

$$m = sB - q\,. \tag{44c}$$

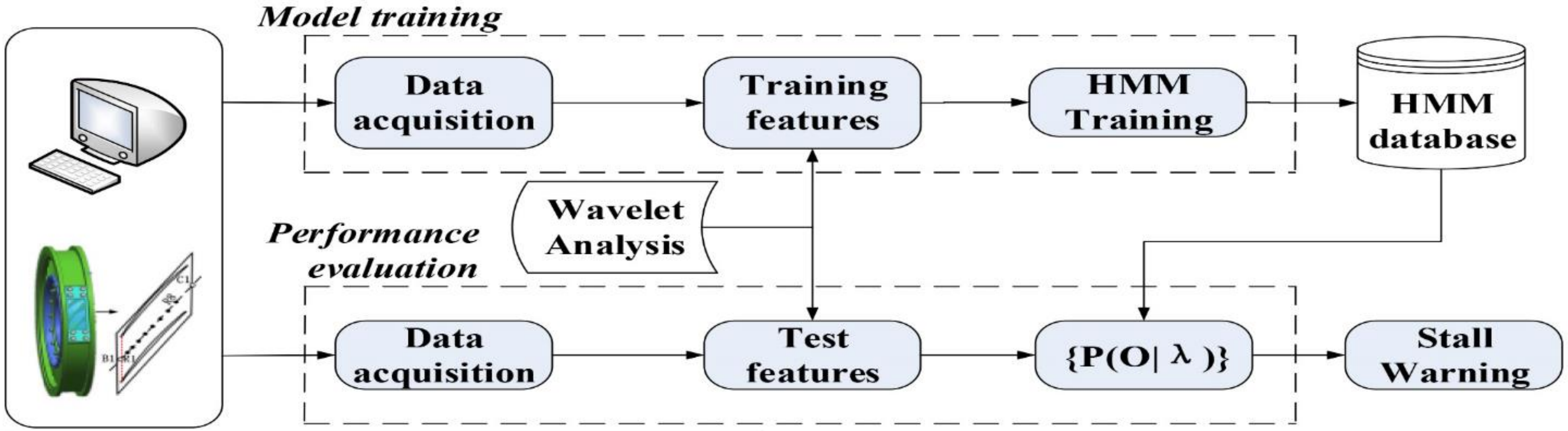


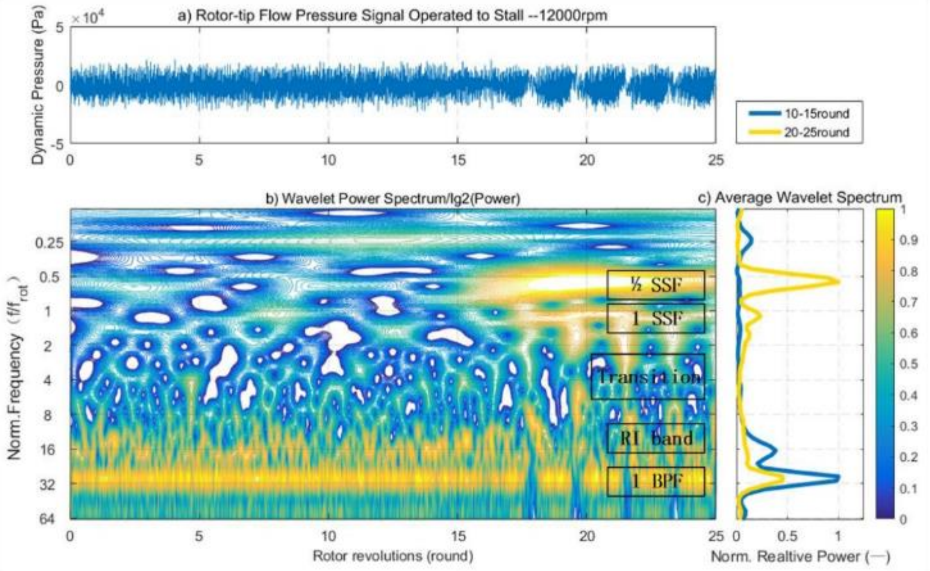
Acoustic Analogy: Tonal Noise

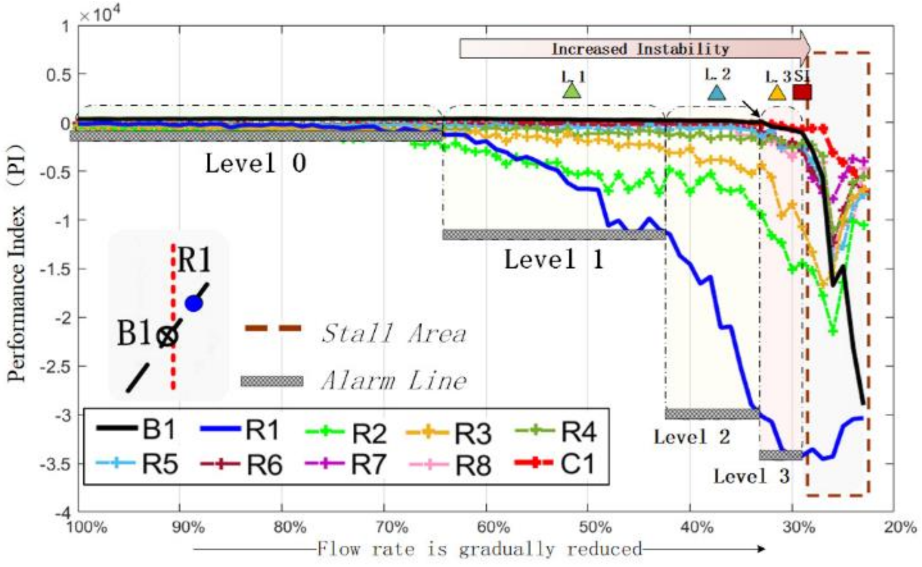
- Extract unsteady CFD data on the surface of the blade;
- Calculate unsteady aerodynamic forces on the surface of the blade;
- Calculate the sound source item (determine the rotating coordinate system, position, etc.);
- Solve the acoustic response equations at the BPF and at each Harmonic position;
- Calculate inlet sound field and post processing



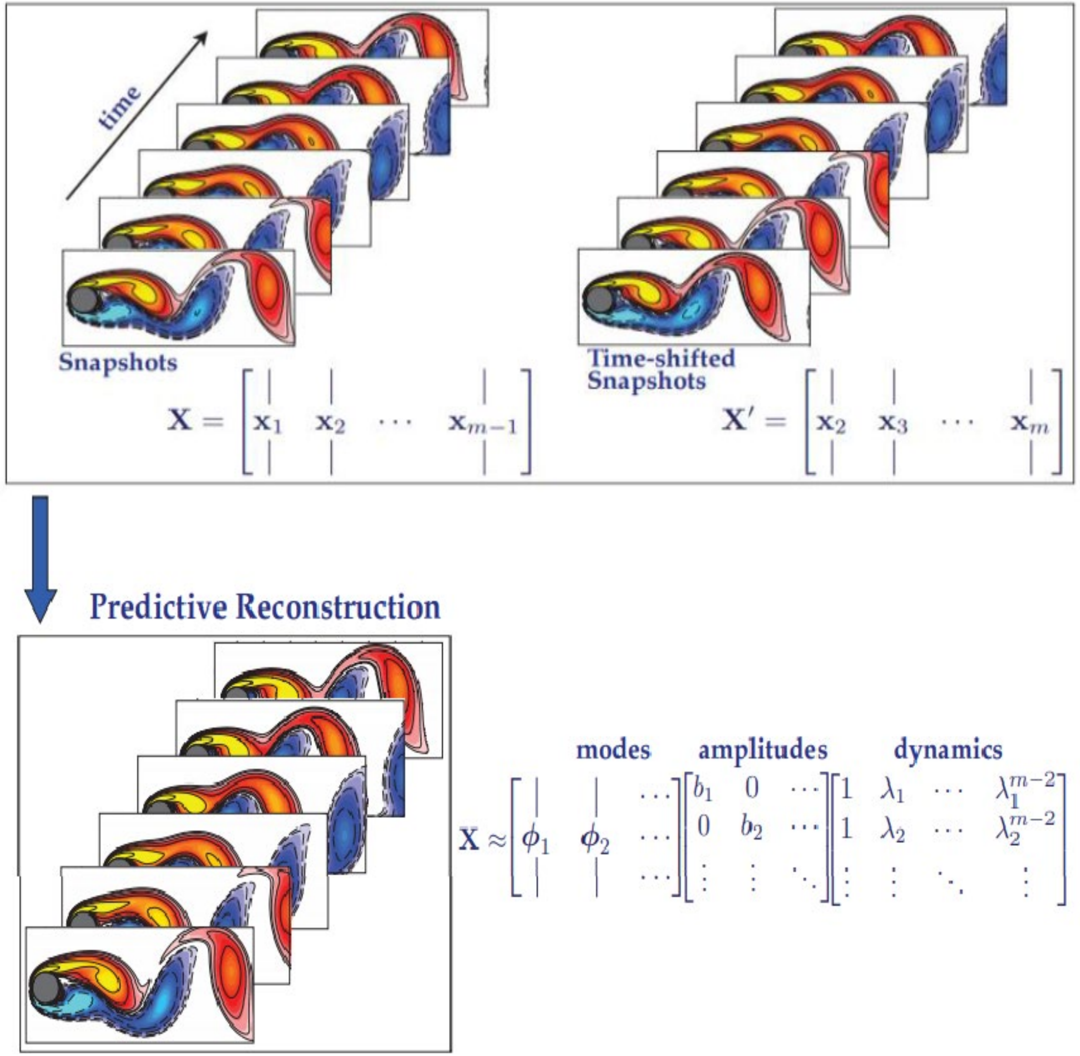
Application: stall warning

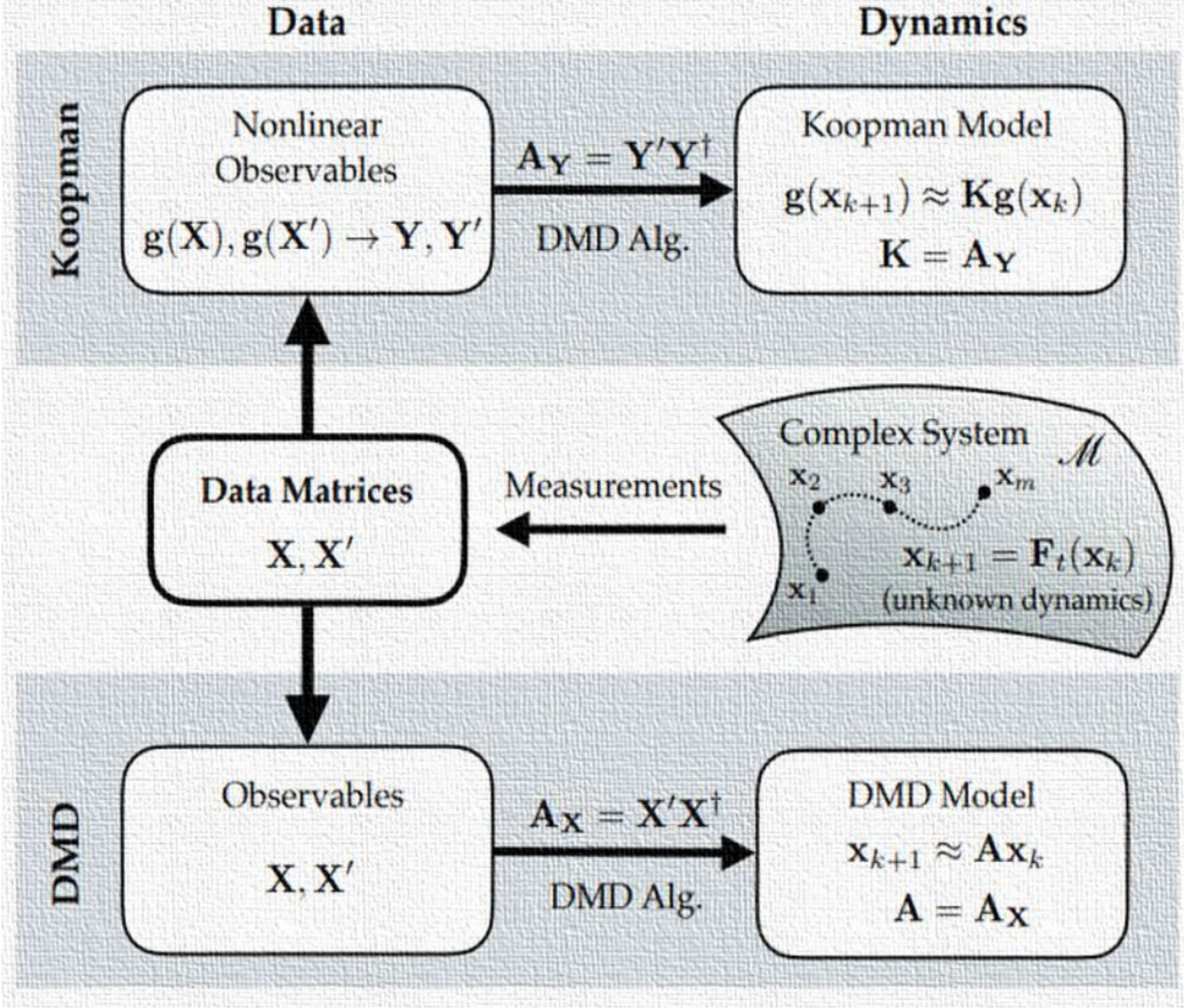




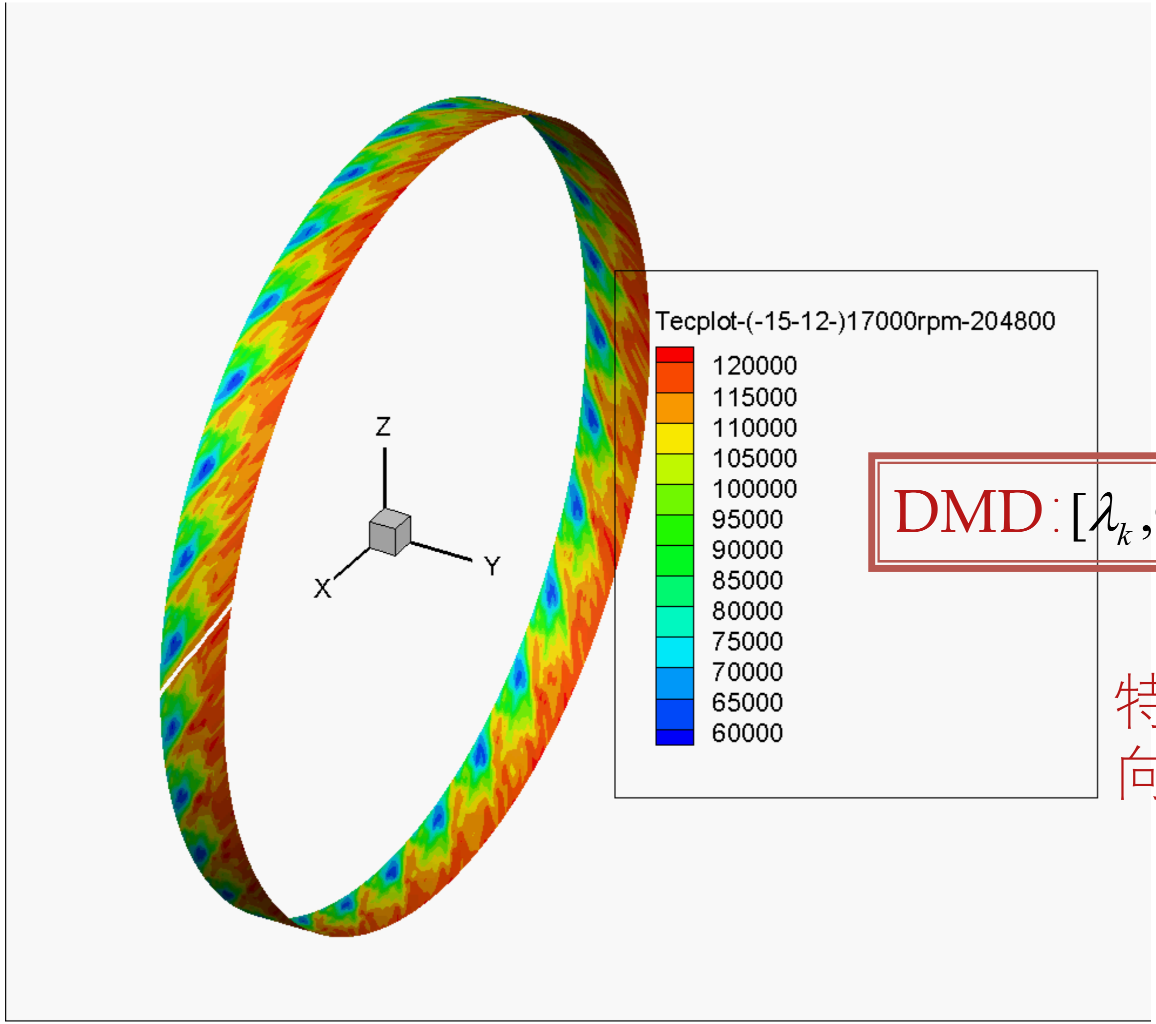


Application: Dynamic mode decomposition



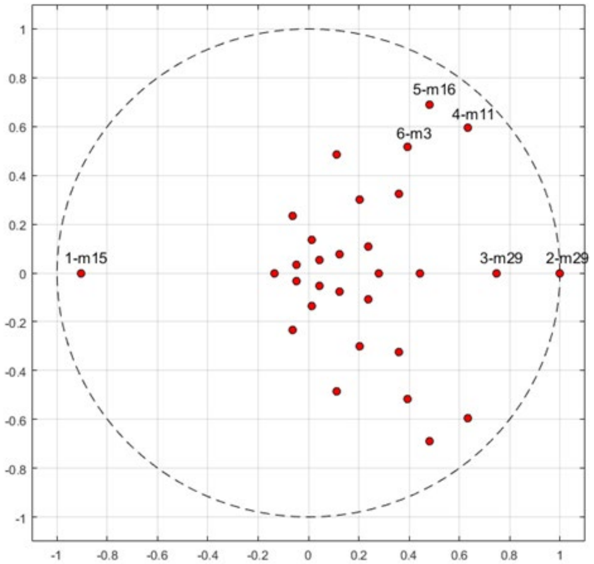


Application: Dynamic mode decomposition

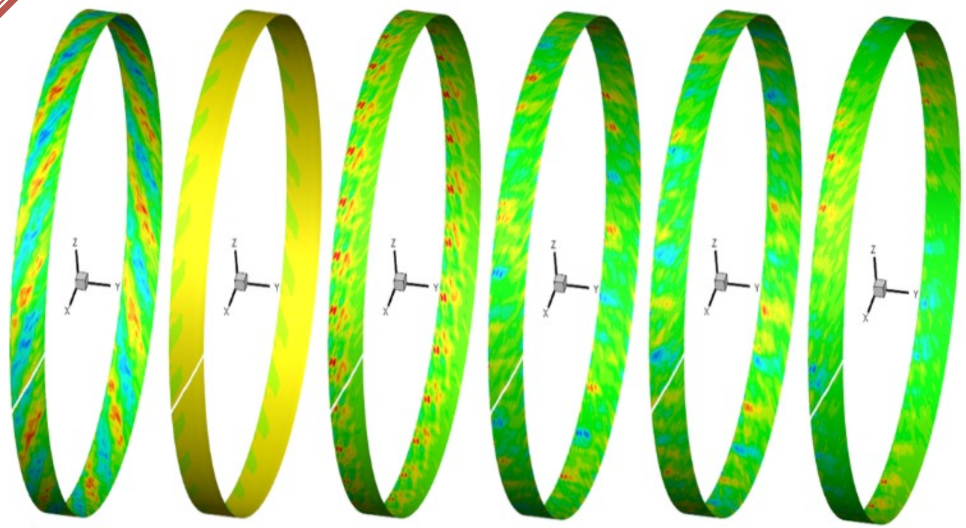


特征值 λ_k

DMD: $[\lambda_k, \Phi]=\mathrm{eig}(\tilde{A})$

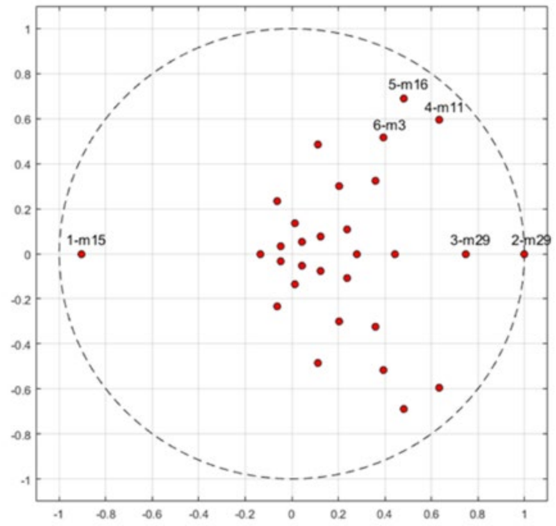


特征向量 Φ



1-m15 2-m29 3-m16 4-m11 5-m16 6-m3

Application: Dynamic mode decomposition

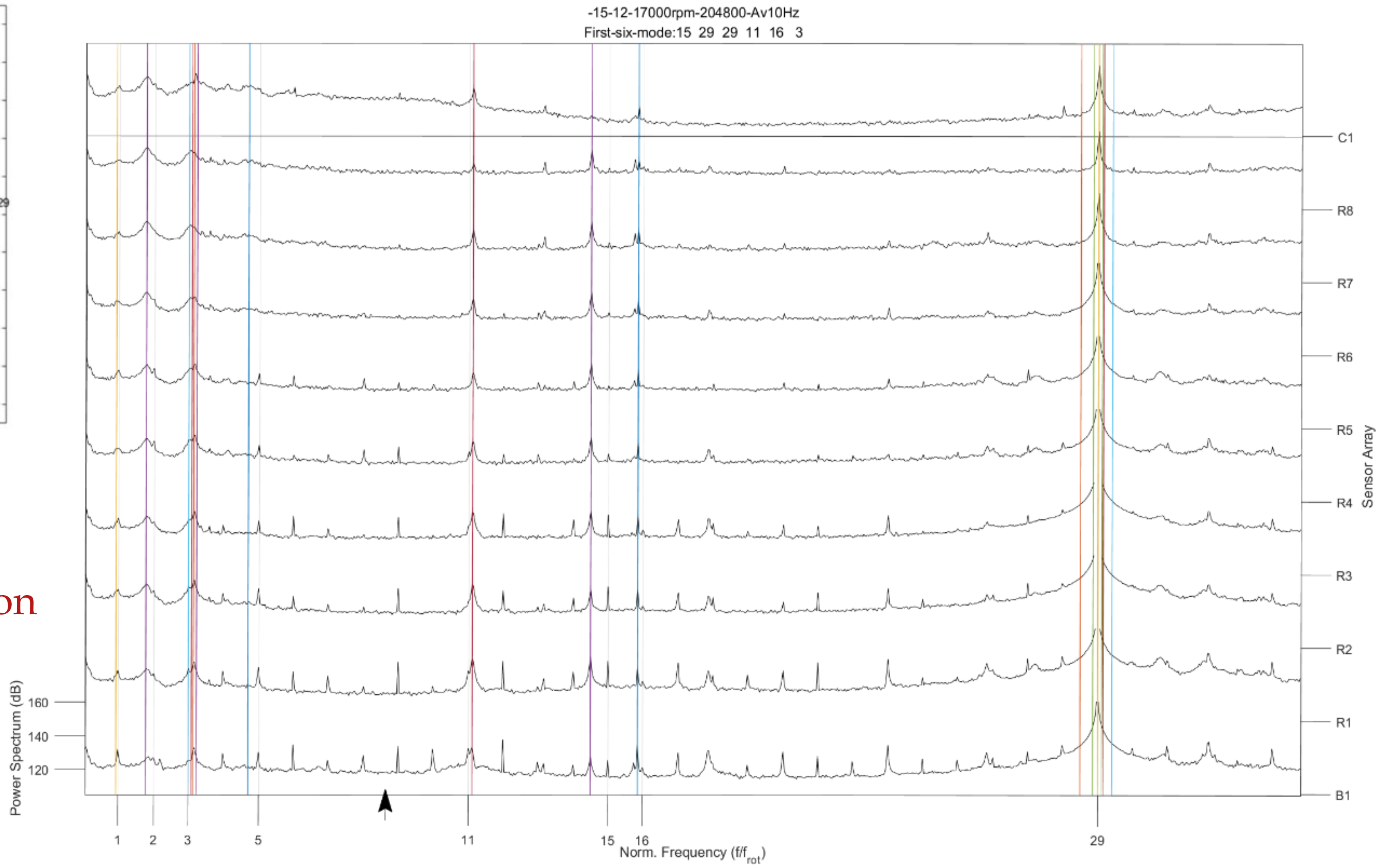


$$w_k = ln(\lambda_k)/T$$

Coordinate system conversion

$$w = w_k \pm m * SSF$$

(Positive and negative corresponding modal rotation Positive and negative directions))



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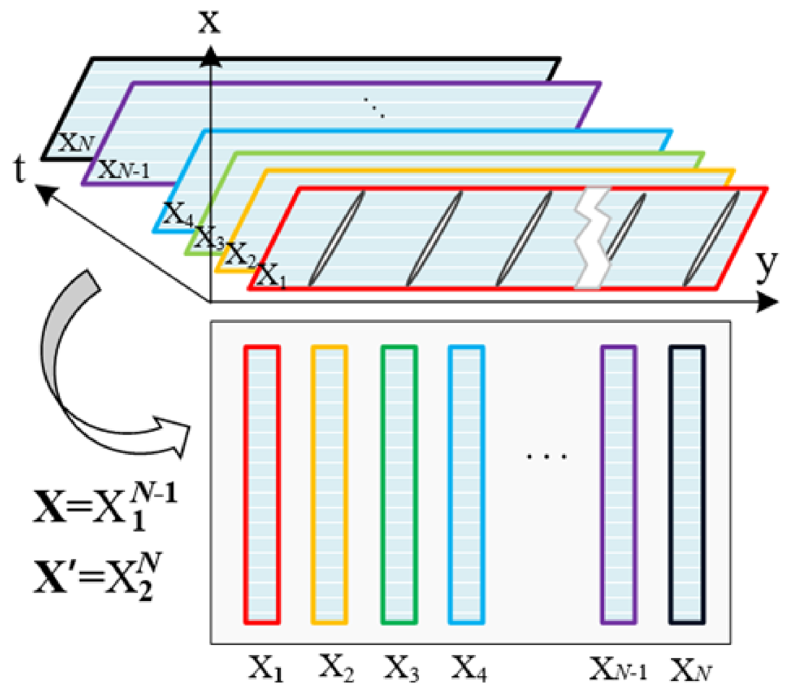
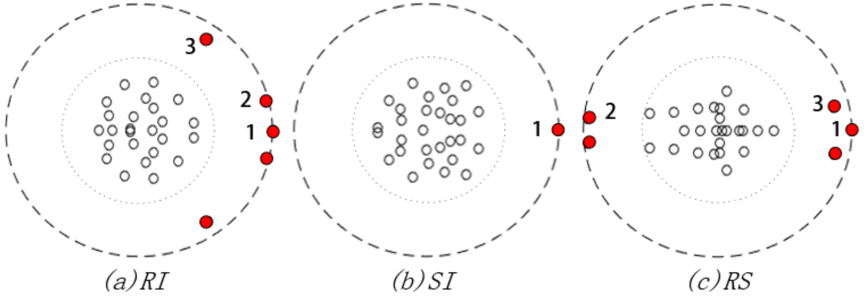
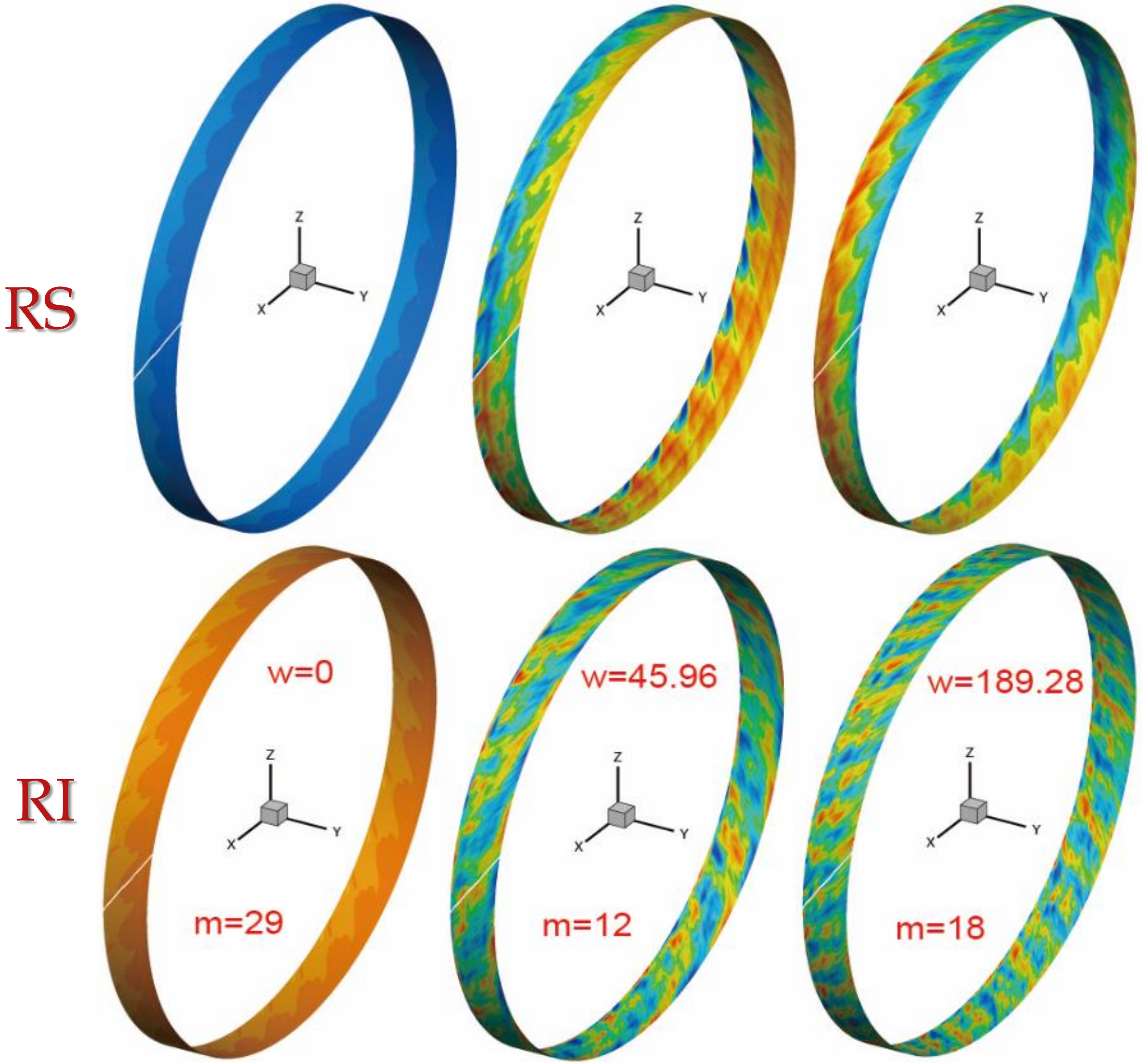
Application: Dynamic mode decomposition







Application: Mode detection

$$\frac{1}{c^2_{\ 0}}\left(\frac{\mathrm{D}}{\mathrm{Dt}}\right)^2 p-\Delta p=0$$

$$p_f(x,r,\theta,t)=\sum_{m=-\infty}^{+\infty}\sum_{n=0}^{+\infty}A_{mnf}E_{mn}(\kappa_{mn}r)\,\mathrm{e}^{\mathrm{i}(2\pi ft+m\theta-\zeta_{mn}x)}$$

$$E_{mn}(\kappa_{mn}r)=C_{mn}[J_m(\kappa_{mn}r)+Q_{mn}Y_m(\kappa_{mn}r)]$$

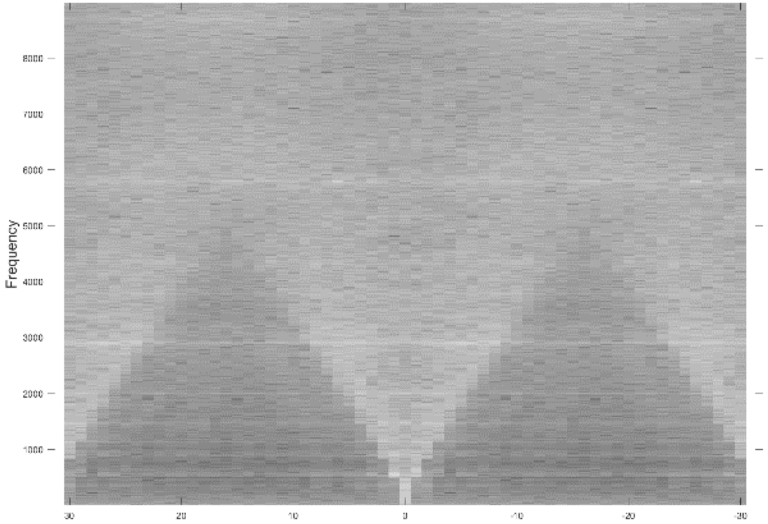
$$p_f(\theta)=\sum_{m=-\infty}^{+\infty}a_{mf}\ \mathrm{e}^{-\mathrm{i}m\theta}$$

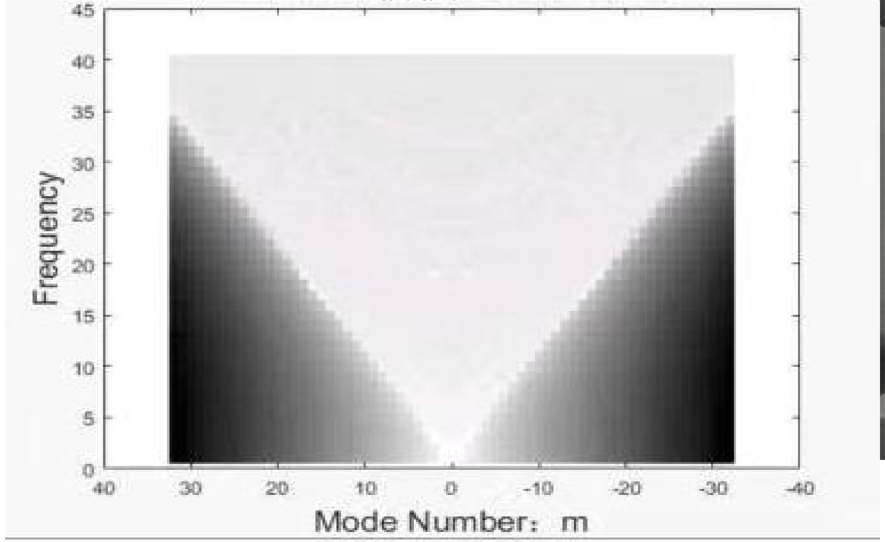
$$a_{mf}=\frac{1}{K}\sum_{k=1}^{K}p_f(\theta_k)e^{im\theta_k}$$

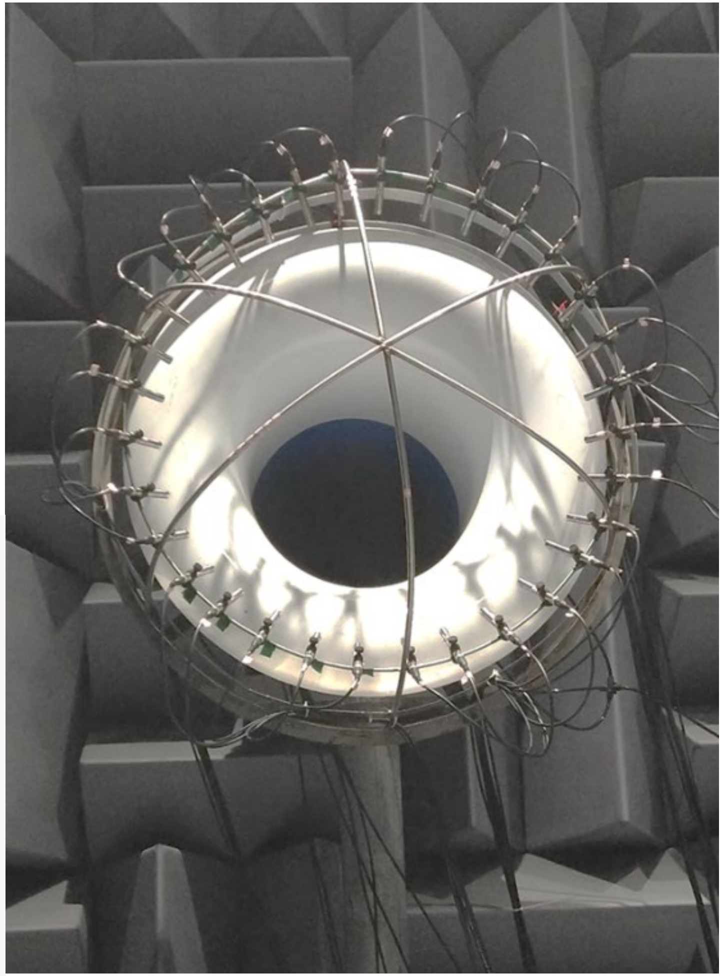
RMS

$$\Gamma_{mf}=\frac{1}{2}\left\langle\left|a_{mf}\right|^2\right\rangle=\frac{1}{2K^2}\left\langle\sum_{k=1}^{K}\sum_{k=1}^{K}p_f(\theta_k)\mathrm{e}^{\mathrm{i}m\theta_k}p_f(\theta_l)\mathrm{e}^{\text{-}\mathrm{i}m\theta_l}\right\rangle=$$

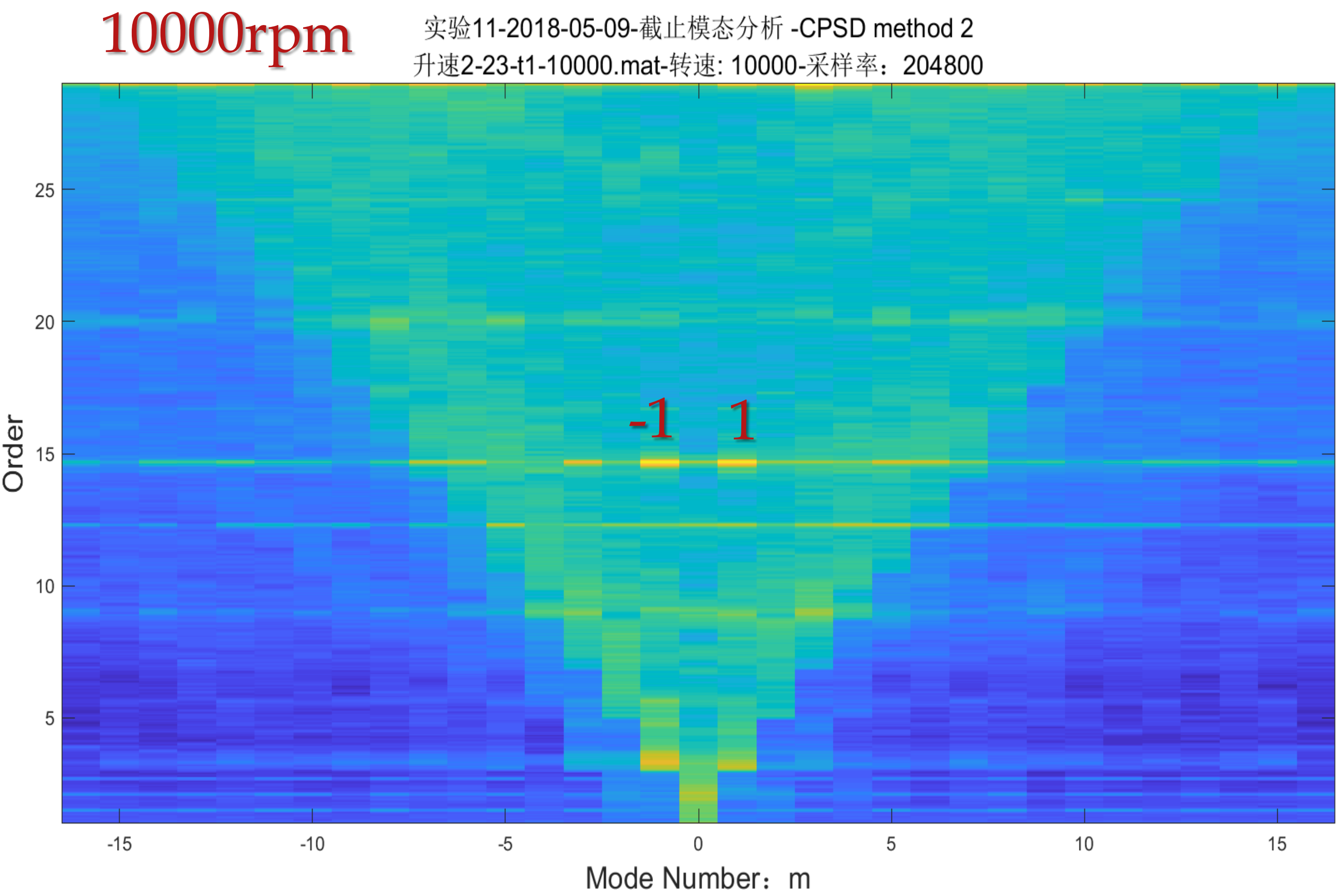
$$\frac{1}{K^2}\sum_{k=1}^{K}\sum_{k=1}^{K}\mathrm{e}^{\mathrm{i}m\theta_k}\left\langle p_f(\theta_k)p_f(\theta_l)^*\right\rangle\mathrm{e}^{\text{-}\mathrm{i}m\theta_l}=\frac{1}{K^2}\sum_{k=1}^{K}\sum_{k=1}^{K}\mathrm{e}^{\mathrm{i}m\theta_k}\mathrm{C}_{kl}\mathrm{e}^{\text{-}\mathrm{i}m\theta_l}$$

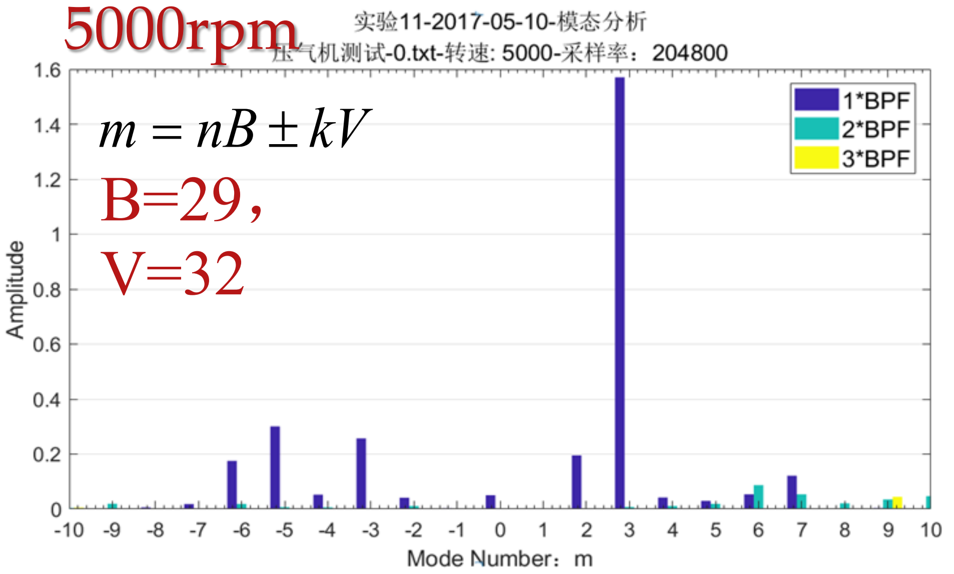


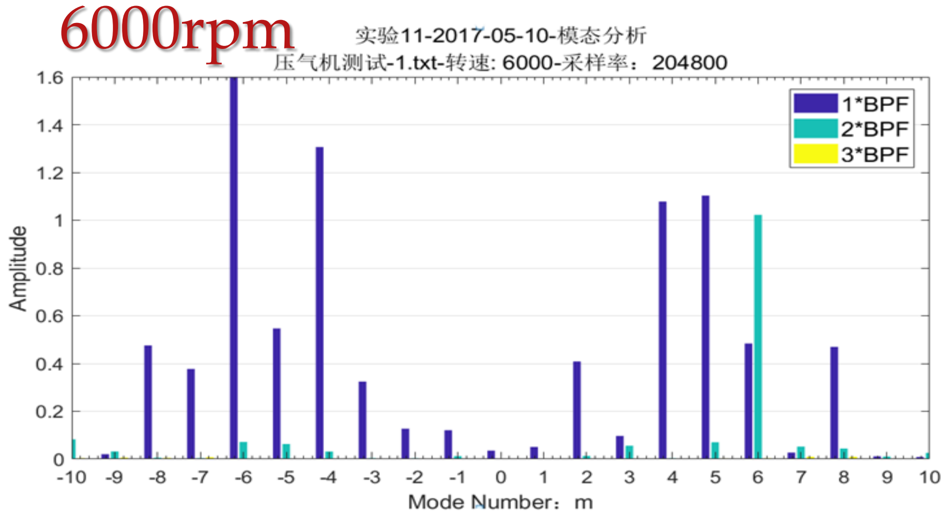




Application: Mode detection







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感谢聆听！