Note:Cyclostaionary Vertification

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A. Radiated sound from a moving body

The FW-H equation is now a widely used starting point for theories of noise generation by moving bodies like propellers in a moving medium, even when turbulence noise is of little or no importance, given as [1]

$$\left[\frac{1}{c_0^2}\frac{D^2}{Dt^2} - \bigtriangledown^2\right]\{p'(\vec{x},t)H(f)\} = \underbrace{\frac{\partial}{\partial x_i \partial x_j}[T_{ij}H(f)]}_{quadrupole} - \underbrace{\frac{\partial}{\partial x_i}L_i\delta(f)}_{loading} + \underbrace{\frac{D}{Dt}[Q\delta(f)]}_{thickness} \tag{1}$$

with

$$\begin{aligned} T_{ij} &= \rho u_i u_j + [(p-p_0) - c_0^2(\rho-\rho_0)]\delta_{ij} - \sigma_{ij} \\ L_i &= \rho u_i[u_n - (v_n - U_{\infty n})] + P_{ij}\hat{n}_j \\ Q &= \rho[u_n - (v_n - U_{\infty n})] + \rho_0(v_n - U_{\infty n}) \end{aligned} \tag{2}$$

H(f) denotes the Heaviside function, which is used to vanish identically within the moving body. The convective derivative $\frac{D}{Dt}$ is defined by

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{U}_\infty \cdot \bigtriangledown \tag{3}$$

where $\vec{U}_\infty$ is the vector of mean flow velocity, and $\bigtriangledown$ the divergence operator.

Combined with the general form the Green's function with impulsive point souces

$$\left[\frac{1}{c_0^2}\frac{D^2}{Dt^2} - \bigtriangledown^2\right] G(\vec{x},t;\vec{y},\tau) = \delta(\vec{x}-\vec{y})\delta(t-\tau) \tag{4}$$

Here we consider the solution of Green's function with arbitrary orientation of $\vec{U}_\infty$ as extend the case of propeller noise to an angle of attack [2].

$$G(\vec{x},t;\vec{y},\tau) = \frac{\delta(\tau - t + R/c_0)}{4\pi R^*} \tag{5}$$

The quantities R^* and R are defined as

$$R^* = \frac{r}{\gamma}\sqrt{1+\gamma^2 M_{\infty r}^2} \tag{6}$$

and

$$R = \gamma^2 (R^* - r M_{\infty r}^2) \tag{7}$$

where $\gamma^2 = 1/(1-M_\infty^2)$ and $M_{\infty r} = \vec{M}_\infty \cdot \hat{r}$ with $\hat{r}$ being the unit vector of the distance between the source position $y(\tau)$ and the observer position x(t), i.e. $r = |x(t) - y(\tau)|$.

Thus, substitute the right hand side of Eq.(1) as the source terms, the solution of FW-H equation can be derived with the property of Green's function

$$\begin{aligned} H(f)p'(\vec{x},t) &= \int_{-\infty}^{t}\int_{v} q(\vec{y},\tau)G(\vec{x},t;\vec{y},\tau)d^3\vec{y}d\tau \\ = \int_{-\infty}^{t}\int_{v}\{\frac{\partial}{\partial x_i \partial x_j}[T_{ij}H(f)] &- \frac{\partial}{\partial x_i}L_i\delta(f) + \frac{D}{Dt}[Q\delta(f)]\}\frac{\delta(\tau - t + R/c_0)}{4\pi R^*}d^3\vec{y}d\tau \end{aligned} \tag{8}$$

B. modeling of propeller noise

For modeling propeller noise, we only concentrate on the loading (2nd term) and thickness (3rd term) contributions, the nonlinear term T_{ij} is ignored. The Eq.(8) is simplified as follows:

$$p'(\vec{x},t) = \int_{-\infty}^{t}\int_{S}\{-\frac{\partial}{\partial x_i}L_i\delta(f) + \frac{D}{Dt}[Q\delta(f)]\}\frac{\delta(\tau - t + R/c_0)}{4\pi R^*}dSd\tau \tag{9}$$

In order to intergrate over $d\tau$, the method of generalized function theory of the Farassat is used:

$$\int_{-\infty}^{t} h(\tau)\delta(g) = [\frac{h(\tau)}{|\partial g/\partial \tau|}]_e \tag{10}$$

where $[\cdot]_e$ is the notion denotes evaluation of the term with $g = 0$, $t = \tau + R/c_0$ as regard time.

The partial differentiation of g with respect to τ yields

$$\frac{\partial g}{\partial \tau} = 1 + \frac{1}{c_0}\frac{\partial R}{\partial \tau} = 1 - M_i\frac{\partial R}{\partial x_i} \tag{11}$$

Here, we only give the final result used for cyclostationarity modeling. More complete derivation can be found in Ghorbaniasl's paper[2].

$$
\begin{aligned}
4\pi p'_L(\vec{x},t) &= \frac{1}{c_0}\frac{\partial}{\partial t}\int_S [\frac{L_R}{R^*(1-M_R)}]_e dS + \int_S [\frac{L_{R^*}}{R^{*2}(1-M_R)}]_e dS \\
4\pi p'_T(\vec{x},t) &= \frac{\partial}{\partial t}\int_S [\frac{(1-M_{\infty R})Q}{R^*(1-M_R)}]_e dS - \int_S [\frac{c_0 M_{\infty R^*} Q}{R^{*2}(1-M_R)}]_e dS
\end{aligned}
\tag{12}
$$

where $L_R = L_i \frac{\partial R}{\partial x_i}$, $L_{R^*} = L_i \frac{\partial R^*}{\partial x_i}$, $M_{\infty R} = M_{\infty i}\frac{\partial R}{\partial x_i}$, $M_{\infty R^*} = M_{\infty i}\frac{\partial R^*}{\partial x_i}$, $M_R = M_i \frac{\partial R}{\partial x_i}$.

The formulation of loading noise p'_L and thickness noise p'_T take acount the convection effect and incidence of mean flow, and is equivalent to the Farassat's formulation 1[3] if the medium is at rest, $\vec{M}_\infty = 0$. The term of $(1-M_R)^{-1}$ refers as the Doppler factor.

References

[1] Ffowcs Williams JE, Hawkings DL "Sound generation by turbulence and surfaces in arbitrary motion". Phil. Trans. R. Soc. Lond. A, **264**, 1969, 321-342.(doi:10.1098/rsta.1969.0031)

[2] Ghorbaniasl G, Lacor C "Amovingmedium formulation for prediction of propeller noise at incidence". J. Sound Vib, **331**, 2012, 117âĂŞ137.(doi:10.1016/j.jsv.2011.08.018)

[3] Farassat F, Myers MK "Multidimensional generalized functions in aeroacoustics and fluid mechanicsâĂŤ-part 1: basic concepts and operations". Int. J. Aeroacoust, **10**(1), 2011, 161âĂŞ200.(doi:10.1260/1475-472X.10.2-3.161)